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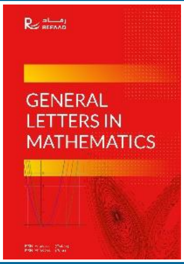

Double SEJI Integral Transform and its Applications of Solution Integral Differential Equations

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Abstract

The aim of this paper is to establish an efficient of a new transform called double SEJI integral transform to solve integral differential equations. Some important properties are proved by this suggested transform with theorem for the partial fractional Caputo derivatives. Finally, we use them to solve applications of some kinds of integral equations by transforming them to algebraic equations and solve by using the giving properties.

Keywords: The double SEJI integral transform, double convolution theorem, Volterra Integral Equation, Volterra Integro- partial differential Equations, Partial Integro- differential Equations.
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1. Introduction

Integral transforms are extremely effective to solve lots of advanced science engineering problems. Many studies have used various integral transforms (Laplace, Sumudu, SEE, SEE complex, Emad- Falih transform, Emad- Sara, etc.) and solved differential and integral equations and their applications. [1, 2, 3, 4, 5, 6, 7].

Some mathematicians are developed transforms to double integral transforms in two dimensional spaces to solve partial differential and integral differential equations such as double Laplace, double Sumudu, double Aboodh, double Kamal, double Mahgoub Transforms, etc. [8, 9, 10, 11, 12], As well as, some of mathematicians applied double transforms for solving fractional partial differential equations [13, 14, 15, 16].

Several of transforms are combined them with exclusive mathematical methods like differential transform approach, homotopy perturbation technique, Adomian decomposition method, and variational iteration method [17, 18, 19, 20, 21, 22, 23, 24, 25, 26].

In this work, we demonstrate the solution of kinds of integral differential equations by applying double SEJI integral transform.

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2. Basic Definition[27]

The SEJI integral transform is a new integral transform similar to the Laplace transform and other integral transforms that are defined in the time domain $t \geq 0$, such as the Sumudu transform, Elzaki transform, integral transform and SEE complex transform.

SEJI integral transform is defined for functions of exponential order. We consider functions in the set C defined by:

$C = \{\psi(x): \exists M, L_1 \text{ and } L_2 \text{ are greater than zero such that } |\psi(x)| < Me^{-iL_j|x|}, \text{ if } x \in (-1)^j \times [0,\infty), j = 1,2, i = \sqrt{-1}\}$.

For a given function in the set C, the constant M must be finite number and L_1, L_2 may be finite or infinite. Then, SEJI integral transform denoted by $T_g^c\{\psi(x)\}$ is defined by the integral equation: (1)

$$T_g^c\{\psi(x)\} = F_g^c(s) = p(s)\int_{x=0}^{\infty} e^{-i\phi(s)x}\,\psi(x)dx,$$

Where $x \geq 0, p(s)$ and $\phi(s)$ are positive real functions, i complex number. [25]

3. The Double SEJI Integral Transform

Definition 3.1: *Let $\psi(x,\Upsilon)$ be an integrable function defined for the variables x and Υ in the first quadrant, $p_1(s), p_2(\kappa) \neq 0$ and $\phi_1(s), \phi_2(\kappa)$ are positive real functions; we define the SEJI double integral transform $T_{2_g}^{c}\{\psi(x,\Upsilon)\}$ by the formula*

$$T_{2_g}^{c}\{\psi(x,\Upsilon)\} = F_{2_g}^{c}(s,\kappa) = p_1(s)\,p_2(\kappa)\int_0^\infty\int_0^\infty e^{-i(\phi_1(s)x+\phi_2(\kappa)\Upsilon)}\,\psi(x,\Upsilon)dxd\Upsilon. \quad (2)$$

Provided that the integral exists for some $\phi_1(s), \phi_2(\kappa)$.

The following formula is the inverse of the Double SEJI integral transform:

$$T_{2_g}^{c\,-1}\left\{F_{2_g}^{c}(s,\kappa)\right\} = \psi(x,\Upsilon) = \frac{1}{2\pi i}\int_{\gamma-i\infty}^{\gamma+i\infty}\frac{i}{p_1(s)}e^{i\phi_1(s)x}ds\left(\frac{1}{2\pi i}\int_{\omega-i\infty}^{\omega+i\infty}\frac{i}{p_2(\kappa)}e^{i\phi_2(\kappa)\Upsilon}F_{2_g}^{c}(s,\kappa)d\kappa\right),$$

Where γ and ω are real constants.

4. Properties of Double SEJI integral transform

4.1. Linearity Property

Let $T_{2_g}^{c}\{\psi(x,\Upsilon)\} = F_{2_{g_1}}^{c}(s,\kappa)$ and $T_{2_g}^{c}\{\Omega(x,\Upsilon)\} = F_{2_{g_1}}^{c}(s,\kappa)$ then for every ν and $\upsilon$are arbitrary constants, then:

$$T_{2_g}^{c}\{\nu\psi(x,\Upsilon) \pm \upsilon\Omega(x,\Upsilon)\} = \nu F_{2_{g_1}}^{c}(s,\kappa) \pm \upsilon F_{2_{g_2}}^{c}(s,\kappa).$$

Proof:

$$T_{2_g}^{c}\{\theta\psi(\chi,\Upsilon) \pm \upsilon\Omega(\chi,\Upsilon)\} = p_1(s)\,p_2(\kappa)\int_0^\infty\int_0^\infty e^{-i(\phi_1(s)x+\phi_2(\kappa)\Upsilon)}[\nu\psi(\chi,\Upsilon) \pm \upsilon\Omega(\chi,\Upsilon)]\,dxd\Upsilon,$$

$$= p_1(s)\,p_2(\kappa)\left(\int_0^\infty\int_0^\infty e^{-i(\phi_1(s)x+\phi_2(\kappa)\Upsilon)}\nu\,\psi(x,\Upsilon)dxd\Upsilon \pm \int_0^\infty\int_0^\infty e^{-i(\phi_1(s)x+\phi_2(\kappa)\Upsilon)}\upsilon\Omega(\chi,\Upsilon)\,dxd\Upsilon\right),$$

$$= p_1(s)\,p_2(\kappa)\left(\nu\int_0^\infty\int_0^\infty e^{-i(\phi_1(s)x+\phi_2(\kappa)\Upsilon)}\,\psi(x,\Upsilon)dxd\Upsilon \right.$$
$$\left.\pm\,\upsilon p_1(s)\,p_2(\kappa)\int_0^\infty\int_0^\infty e^{-i(\phi_1(s)x+\phi_2(\kappa)\Upsilon)}\Omega(\chi,\Upsilon)\,dxd\Upsilon\right),$$

$$= \nu p_1(s)\,p_2(\kappa)\left(\int_0^\infty\int_0^\infty e^{-i(\phi_1(s)x+\phi_2(\kappa)\Upsilon)}\,\psi(x,\Upsilon)dxd\Upsilon\right)$$
$$\pm\,\upsilon p_1(s)\,p_2(\kappa)\left(\int_0^\infty\int_0^\infty e^{-i(\phi_1(s)x+\phi_2(\kappa)\Upsilon)}\,\Omega(\chi,\Upsilon)dxd\Upsilon\right),$$

$$T_{2_g}^{c}\{\nu\psi(x,\Upsilon) \pm \upsilon\Omega(x,\Upsilon)\} = \nu F_{2_{g_1}}^{c}(s,\kappa) \pm \upsilon F_{2_{g_2}}^{c}(s,\kappa). \qquad \blacksquare$$

4.2 Shifting Property:

If $T_{2_g}^{c}\{\psi(x,\Upsilon)\} = F_{2_g}^{c}(s,\kappa)$ then

$$T_{2_g}^{c}\{e^{-(\nu x+\upsilon\Upsilon)}\psi(x,\Upsilon)\} = F_{2_g}^{c}[(i\phi_1(s)+\nu),(i\phi_2(\kappa)+\upsilon)].$$

Where $\nu, \upsilon \in R$.

Proof:

$$T_{2_g}^{c}\{\psi(x,\Upsilon)\} = F_{2_g}^{c}(\vartheta,\kappa) = p_1(s)\,p_2(\kappa)\int_0^\infty\int_0^\infty e^{-i(\phi_1(s)x+\phi_2(\kappa)\Upsilon)}\,\psi(x,\Upsilon)dxd\Upsilon.$$

Then the double SEJI integral transform of $e^{-(\nu x+\upsilon \Upsilon)}\psi(x,\Upsilon)$ is:

$$T_{2_g}^{\;c}\{e^{-(\nu x+\upsilon \Upsilon)}\psi(x,\Upsilon)\} = p_1(s)\,p_2(\kappa)\int_0^\infty\int_0^\infty e^{-i(\phi_1(s)x+\phi_2(\kappa)\Upsilon)}e^{-(\nu x+\upsilon\Upsilon)}\,\psi(x,\Upsilon)dxd\Upsilon,$$

$$= p_1(s)\,p_2(\kappa)\int_0^\infty\int_0^\infty e^{-[(i\phi_1(s)+\nu)\chi+(i\phi_2(\kappa)+\upsilon)\Upsilon]}\,\psi(x,\Upsilon)dxd\Upsilon,$$

$$T_{2_g}^{\;c}\{e^{-(\nu x+\upsilon \Upsilon)}\psi(\chi,\Upsilon)\} = F_{2_g}^{\;c}[(i\phi_1(s)+\nu),(i\phi_2(\kappa)+\upsilon)]. \qquad \blacksquare$$

4.3 Change of Scale Property:

If $T_{2_g}^{\;c}\{\psi(x,\Upsilon)\} = F_{2_g}^{\;c}(s,\kappa)$then

$$T_{2_g}^{\;c}\{f(\nu x,\upsilon\Upsilon)\} = \frac{1}{\nu\upsilon}F_{2_g}^{\;c}[\frac{\phi_1(s)}{\nu},\frac{\phi_2(\kappa)}{\upsilon}].$$

Where $\nu,\upsilon \in R$.

Proof:

$$T_{2_g}^{\;c}\{\psi(\nu x,\upsilon\Upsilon)\} = p_1(s)\,p_2(\kappa)\int_0^\infty\int_0^\infty e^{-i(\phi_1(s)x+\phi_2(\kappa)\Upsilon)}\,\psi(\nu x,\upsilon\Upsilon)dxd\Upsilon,$$

Substituting $\nu x = \mu$ and $\upsilon\Upsilon = \varpi$ in the previous equation,
Hence, as $x\to 0,\mu\to 0$ and $\Upsilon\to 0,\varpi\to 0$, also
$x\to\infty,\mu\to\infty$ and $\Upsilon\to\infty,\varpi\to\infty$.
$\nu dx = d\mu$ and $\upsilon d\Upsilon = d\varpi$, then $dx = \frac{d\mu}{\nu}$ and $d\Upsilon = \frac{d\varpi}{\upsilon}$.

$$T_{2_g}^{\;c}\{\psi(\nu x,\upsilon\Upsilon)\} = p_1(s)\,p_2(\kappa)\int_0^\infty\int_0^\infty e^{-i\left(\phi_1(s)\frac{\mu}{\nu}+\phi_2(\kappa)\frac{\varpi}{\upsilon}\right)}\psi(\mu,\varpi)\frac{d\mu}{\nu}\frac{d\varpi}{\upsilon},$$

$$= \frac{1}{\nu\upsilon}\left(p_1(s)\,p_2(\kappa)\int_0^\infty\int_0^\infty e^{-i\left(\frac{\phi_1(s)}{\nu}\mu+\frac{\phi_2(\kappa)}{\upsilon}\varpi\right)}\psi(\mu,\varpi)d\mu d\varpi\right),$$

then

$$T_{2_g}^{\;c}\{\psi(\nu x,\upsilon\Upsilon)\} = \frac{1}{\nu\upsilon}F_{2_g}^{\;c}[\frac{\phi_1(s)}{\nu},\frac{\phi_2(\kappa)}{\upsilon}]. \qquad \blacksquare$$

4.4 Double Convolution Theorem:

Suppose that $\psi(x,\Upsilon)$, $\Omega(x,\Upsilon)$ are continuous functions with two variables, then the double convolution of $\psi(x,\Upsilon)$, $\Omega(x,\Upsilon)$is written as:

$$\psi(x,\Upsilon) ** \Omega(x,\Upsilon) = \int_0^\Upsilon\int_0^x \psi(x-\tau,\Upsilon-\mu)\,\Omega(x,\Upsilon)d\tau d\mu.$$

Where $(**)$ denotes the double convolution for x, Υ.

Theorem (4.1): Let $F_{2_{g_1}}^{\;c}(s,\kappa)$, $F_{2_{g_2}}^{\;c}(s,\kappa)$ be SEJI double integral transform of the functions $\psi(x,\Upsilon)$, $\Omega(x,\Upsilon)$ respectively, $\Theta_1(\vartheta)\Theta_2(\kappa)\neq 0,\forall\vartheta,\kappa>0$, then

$$T_{2_g}^{\;c}\{\psi(x,\Upsilon) ** \Omega(x,\Upsilon)\} = \frac{1}{\Theta_1(\vartheta)\Theta_2(\kappa)}F_{2_{g_1}}^{\;c}(s,r)F_{2_{g_2}}^{\;c}(s,r).$$

Proof: we have

$$T_{2_g}^{\;c}\{\psi(x,\Upsilon) ** \Omega(x,\Upsilon)\} = p_1(s)\,p_2(\kappa)\int_0^\infty\int_0^\infty e^{-i(\phi_1(s)x+\phi_2(\kappa)\Upsilon)}\left(\int_0^\Upsilon\int_0^x \psi(x-\tau,\Upsilon-\mu)\,\Omega(\tau,\mu)d\tau d\mu\right)dxd\Upsilon,$$

Substituting $\nu = x-\tau,\upsilon = \Upsilon-\mu$ and letting x,Υ to ∞, we get

$$T_{2_g}^{\;c}\{\psi(x,\Upsilon) ** \Omega(x,\Upsilon)\} = p_1(s)\,p_2(\kappa)\int_0^\infty\int_0^\infty\left(\int_{-\tau}^\infty\int_{-\tau}^\infty \psi(\nu,\upsilon)\Omega(\tau,\mu)d\tau d\mu\right)e^{-i\left(\phi_1(s)(\nu+\tau)+\phi_2(\kappa)(\upsilon+\mu)\right)}d\nu d\upsilon,$$

$$= p_1(s)\int_0^\infty\int_0^\infty \psi(\nu,\upsilon)\left(p_2(\kappa)\int_{-\tau}^\infty\int_{-\tau}^\infty e^{-i(\phi_1(s)\tau+\phi_2(\kappa)\mu)}\,\Omega(\tau,\mu)d\tau d\mu\right)e^{-i(\phi_1(s)\nu+\phi_2(\kappa)\upsilon)}d\nu d\upsilon,$$

$$= \int_0^\infty\int_0^\infty \psi(\nu,\upsilon)\left(p_1(s)\,p_2(\kappa)\int_0^\infty\int_0^\infty e^{-i(\phi_1(s)\tau+\phi_2(\kappa)\mu)}\,\Omega(\tau,\mu)d\tau d\mu\right)e^{-i(\phi_1(s)\nu+\phi_2(\kappa)\upsilon)}d\nu d\upsilon,$$

Because both functions $\psi(x,\Upsilon)$ and $\Omega(x,\Upsilon)$ have zero negative values. Therefore, it yields that

$$T_{2_g}^{\;c}\{\psi(x,\Upsilon) ** \Omega(x,\Upsilon)\} = T_{2_g}^{\;c}\{\Omega(x,\Upsilon)\}\int_0^\infty\int_0^\infty \psi(\nu,\upsilon)\,e^{-i(\phi_1(s)\nu+\phi_2(\kappa)\upsilon)}d\nu d\upsilon,$$

$$T_{2_g}^{\;c}\{\psi(x,\Upsilon) ** \Omega(x,\Upsilon)\} = \frac{1}{p_1(s)\,p_2(\kappa)}T_{2_g}^{\;c}\{\psi(x,\Upsilon)\}T_{2_g}^{\;c}\{\Omega(x,\Upsilon)\},$$

$$T_{2g}^{c}\{\psi(x,\Upsilon) ** \Omega(x,\Upsilon)\} = \frac{1}{p_1(s)\, p_2(\kappa)} F_{2g_1}^{c}(s,\kappa) F_{2g_2}^{c}(s,\kappa). \qquad \blacksquare$$

5. The Double SEJI Integral Transform for some Fundamental Functions

In this section, we shall derive some elementary functions by using Double SEJI integral transform.

Formula 5.1.

$\psi(x,\Upsilon) = 1, x, \Upsilon > 0,$

$$T_{2g}^{c}\{1\} = p_1(s)\, p_2(\kappa) \int_0^\infty \int_0^\infty e^{-i(\phi_1(s)x+\phi_2(\kappa)\Upsilon)}\, dx d\Upsilon,$$

$$= p_1(s)\, p_2(\kappa) \int_0^\infty e^{-i\phi_2(\kappa)\Upsilon} \left(\int_0^\infty e^{-i\phi_1(s)x}\, dx \right) d\Upsilon,$$

$$= p_1(s)\, p_2(\kappa) \int_0^\infty e^{-i\phi_2(\kappa)\Upsilon} \left(\frac{-1}{i\phi_1(s)} e^{-i\phi_1(s)x} \Big|_0^\infty \right) d\Upsilon,$$

$$= \frac{p_1(s)\, p_2(\kappa)}{i\phi_1(s)} \frac{-1}{i\phi_2(\kappa)} \left(e^{-i\phi_2(\kappa)\Upsilon} \Big|_0^\infty \right),$$

$$T_{2g}^{c}\{1\} = \frac{-p_1(s)\, p_2(\kappa)}{\phi_1(s)\phi_2(\kappa)}.$$

Formula 5.2.

$\psi(x,\Upsilon) = e^{\nu x+\upsilon\Upsilon}, \nu, \upsilon \in R,$

$$T_{2g}^{c}\{e^{\nu x+\upsilon\Upsilon}\} = p_1(s)\, p_2(\kappa) \int_0^\infty \int_0^\infty e^{-i(\phi_1(s)x+\phi_2(\kappa)\Upsilon)} e^{\nu x+\upsilon\Upsilon}\, dx d\Upsilon,$$

$$= p_1(s)\, p_2(\kappa) \int_0^\infty e^{-(i\phi_2(\kappa)-\upsilon)\Upsilon} \left(\int_0^\infty e^{-(i\phi_1(s)-\nu)x}\, dx \right) d\Upsilon,$$

$$T_{2g}^{c}\{e^{\nu x+\upsilon\Upsilon}\} = p_1(s)\, p_2(\kappa) \int_0^\infty e^{-(i\phi_2(\kappa)-\upsilon)\Upsilon} \left(\frac{-1}{i\phi_1(s)-\nu} e^{e^{-(i\phi_1(s)-\nu)x}} \Big|_0^\infty \right) d\Upsilon,$$

$$= \frac{\Theta_1(\vartheta)\Theta_2(\kappa)}{i\phi_1(s)-\nu} \left(e^{-(i\phi_2(\kappa)-\upsilon)\Upsilon} \Big|_0^\infty \right),$$

$$= \frac{p_1(s)\, p_2(\kappa)}{(i\phi_1(s)-\nu)(i\phi_2(\kappa)-\upsilon)},$$

$$T_{2g}^{c}\{e^{\nu x+\upsilon\Upsilon}\} = \frac{p_1(s)\, p_2(\kappa)}{(\nu^2+[\phi_1(s)]^2)(\upsilon^2+[\phi_2(\kappa)]^2)} \big(\nu + i\phi_1(s)\big)\big(\upsilon + i\phi_2(\kappa)\big).$$

Formula 5.3.

$\psi(x,\Upsilon) = e^{i(\nu x+\upsilon\Upsilon)}, \nu, \upsilon \in R,$

$$T_{2g}^{c}\{e^{i(\nu x+\upsilon\Upsilon)}\} = p_1(s)\, p_2(\kappa) \int_0^\infty \int_0^\infty e^{-i(\phi_1(s)x+\phi_2(\kappa)\Upsilon)}\, e^{i(\nu x+\upsilon\Upsilon)} dx d\Upsilon,$$

$$= p_1(s)\, p_2(\kappa) \int_0^\infty e^{-i(\phi_2(\kappa)-\upsilon)\Upsilon} \left(\int_0^\infty e^{-i(\phi_1(s)-\nu)x}\, dx \right) d\Upsilon,$$

$$T_{2g}^{c}\{e^{i(\nu x+\upsilon\Upsilon)}\} = \frac{-p_1(s)\, p_2(\kappa)}{(\phi_1(s)-\nu)(\phi_2(\kappa)-\upsilon)}.$$

Formula 5.4.

$$\psi(x,\Upsilon) = \sin(\nu x+\upsilon\Upsilon), \nu, \upsilon \in R,$$

$$T_{2g}^{c}\{\sin(\nu x+\upsilon\Upsilon)\} = p_1(s)\, p_2(\kappa) \int_0^\infty \int_0^\infty e^{-i(\phi_1(s)x+\phi_2(\kappa)\Upsilon)} \sin(\nu x+\upsilon\Upsilon)\, dx d\Upsilon,$$

$$= p_1(s)\, p_2(\kappa) \int_0^\infty \int_0^\infty e^{-i(\phi_1(s)x+\phi_2(\kappa)\Upsilon)} \left[\frac{e^{i(\nu x+\upsilon\Upsilon)} - e^{-i(\nu x+\upsilon\Upsilon)}}{2i} \right] dx d\Upsilon,$$

$$= \tfrac{1}{2i} p_1(s)\, p_2(\kappa) \Big(\int_0^\infty \int_0^\infty e^{-i(\phi_1(s)x+\phi_2(\kappa)\Upsilon)}\, e^{i(\nu x+\upsilon\Upsilon)} dx d\Upsilon - \int_0^\infty \int_0^\infty e^{-i(\phi_1(s)x+\phi_2(\kappa)\Upsilon)}\, e^{-i(\nu x+\upsilon\Upsilon)} dx d\Upsilon \Big),$$

$$= \tfrac{1}{2i} p_1(s)\, p_2(\kappa) \Big(\int_0^\infty \int_0^\infty e^{-i(\phi_1(s)-\nu)x}\, e^{-i(\phi_2(\kappa)-\upsilon)\Upsilon} dx d\Upsilon - \int_0^\infty \int_0^\infty e^{-i(\phi_1(s)+\nu)x}\, e^{-i(\phi_2(\kappa)+\upsilon)\Upsilon} dx d\Upsilon \Big),$$

$$T_{2g}^{c}\{\sin(\nu x+\upsilon\Upsilon\Upsilon)\} = \frac{1}{2i} \left[\frac{-p_1(s)\, p_2(\kappa)}{(\phi_1(s)-\nu)(\phi_2(\kappa)-\upsilon)} + \frac{p_1(s)\, p_2(\kappa)}{(\phi_1(s)+\nu)(\phi_2(\kappa)+\upsilon)} \right]$$

$$T_{2g}^{c}\{\sin(\nu x+\upsilon\Upsilon)\} = \frac{-ip_1(s)\, p_2(\kappa)[\upsilon\phi_1(s) + \nu\phi_2(\kappa)]}{([\phi_1(s)]^2-\nu^2)([\phi_2(\kappa)]^2-\upsilon^2)}.$$

Formula 5.5.
$\psi(x,\Upsilon) = cos\ (\nu x + \upsilon\Upsilon)\ , \nu, \upsilon \in R,$

$$T_{2}{}_{g}^{c}\{cos\ (\nu x + \upsilon\Upsilon)\ \} = p_1(s)\ p_2(\kappa) \int_0^\infty \int_0^\infty e^{-i(\phi_1(s)x+\phi_2(\kappa)\Upsilon)}\ cos\ (\nu x + \upsilon\Upsilon)\ dxd\Upsilon,$$
$$= p_1(s)\ p_2(\kappa) \int_0^\infty \int_0^\infty e^{-i(\phi_1(s)x+\phi_2(\kappa)\Upsilon)} \left[\frac{e^{i(\nu x+\upsilon\Upsilon)} + e^{-i(\nu x+\upsilon\Upsilon)}}{2}\right] dxd\Upsilon,$$
$$= \frac{1}{2}\Theta_1(\vartheta)\Theta_2(\kappa) \left(\int_0^\infty \int_0^\infty e^{-i(\phi_1(s)x+\phi_2(\kappa)\Upsilon)}\ e^{i(\nu x+\upsilon\Upsilon)} dxd\Upsilon \right.$$
$$\left. + \int_0^\infty \int_0^\infty e^{-i(\phi_1(s)x+\phi_2(\kappa)\Upsilon)}\ e^{-i(\nu x+\upsilon\Upsilon)} dxd\Upsilon \right),$$
$$= \frac{1}{2}p_1(s)\ p_2(\kappa) \left(\int_0^\infty \int_0^\infty e^{-i(\phi_1(s)-\nu)x}\ e^{-i(\phi_2(\kappa)-\upsilon)\Upsilon} dxd\Upsilon + \int_0^\infty \int_0^\infty e^{-i(\phi_1(s)+\nu)x}\ e^{-i(\phi_2(\kappa)+\upsilon)\Upsilon} dxd\Upsilon \right),$$
$$= \frac{1}{2}\left[\frac{-p_1(s)\ p_2(\kappa)}{(\phi_1(s)-\nu)(\phi_2(\kappa)-\upsilon)} - \frac{p_1(s)\ p_2(\kappa)}{(\phi_1(s)+\nu)(\phi_2(\kappa)+\upsilon)}\right]$$
$$T_{2}{}_{g}^{c}\{cos\ (\nu\chi + \upsilon\Upsilon)\ \} = \frac{-p_1(s)\ p_2(\kappa)[\phi_1(s)\phi_2(\kappa) + \nu\upsilon]}{([\phi_1(s)]^2 - \nu^2)([\phi_2(\kappa)]^2 - \upsilon^2)}.$$

Formula 5.6.
$\psi(x,\Upsilon) = sinh\ (\nu x + \upsilon\Upsilon)\ , \nu, \upsilon \in R,$

$$T_{2}{}_{g}^{c}\{sinh\ (\nu x + \upsilon\Upsilon)\} = p_1(s)\ p_2(\kappa) \int_0^\infty \int_0^\infty e^{-i(\phi_1(s)x+\phi_2(\kappa)\Upsilon)}\ sinh\ (\nu x + \upsilon\Upsilon)\ dxd\Upsilon,$$
$$= p_1(s)\ p_2(\kappa) \int_0^\infty \int_0^\infty e^{-i(\phi_1(s)x+\phi_2(\kappa)\Upsilon)} \left[\frac{e^{(\nu x+\upsilon\Upsilon)} - e^{-(\nu x+\upsilon\Upsilon)}}{2}\right] dxd\Upsilon,$$
$$= \frac{1}{2}p_1(s)\ p_2(\kappa) \left(\int_0^\infty \int_0^\infty e^{-i(\phi_1(s)x+\phi_2(\kappa)\Upsilon)}\ e^{(\nu x+\upsilon\Upsilon)} dxd\Upsilon - \int_0^\infty \int_0^\infty e^{-i(\phi_1(s)x+\phi_2(\kappa)\Upsilon)}\ e^{-(\nu x+\upsilon\Upsilon)} dxd\Upsilon \right),$$
$$= \frac{1}{2}p_1(s)\ p_2(\kappa) \left(\int_0^\infty \int_0^\infty e^{-(\phi_1(s)-\nu)x}\ e^{-(i\phi_2(\kappa)-\upsilon)\Upsilon} dxd\Upsilon - \int_0^\infty \int_0^\infty e^{-(\phi_1(s)+\nu)x}\ e^{-(i\phi_2(\kappa)+\upsilon)\Upsilon} dxd\Upsilon \right),$$
$$= \frac{1}{2}\left[\frac{-p_1(s)\ p_2(\kappa)}{(i\phi_1(s)-\nu)(i\phi_2(\kappa)-\upsilon)} - \frac{p_1(s)\ p_2(\kappa)}{(i\phi_1(s)+\nu)(i\phi_2(\kappa)+\upsilon)}\right],$$
$$T_{2}{}_{g}^{c}\{sinh\ (\nu x + \upsilon\Upsilon)\ \} = \frac{-ip_1(s)\ p_2(\kappa)[\upsilon\phi_1(s) + \nu\phi_2(\kappa)]}{([\phi_1(s)]^2 + \nu^2)([\phi_2(\kappa)]^2 + \upsilon^2)}$$

Formula 5.7.
$\psi(\chi,\Upsilon) = cosh\ (\nu\chi + \upsilon\Upsilon)\ , \nu, \upsilon \in R,$

$$T_{2}{}_{g}^{c}\{cosh\ (\nu\chi + \upsilon\Upsilon)\ \} = p_1(s)\ p_2(\kappa) \int_0^\infty \int_0^\infty e^{-i(\phi_1(s)x+\phi_2(\kappa)\Upsilon)}\ cosh\ (\nu x + \upsilon\Upsilon)\ dxd\Upsilon,$$
$$= p_1(s)\ p_2(\kappa) \int_0^\infty \int_0^\infty e^{-i(\phi_1(s)x+\phi_2(\kappa)\Upsilon)} \left[\frac{e^{(\nu x+\upsilon\Upsilon)} + e^{-(\nu x+\upsilon\Upsilon)}}{2}\right] dxd\Upsilon,$$
$$= \frac{1}{2}p_1(s)\ p_2(\kappa) \left(\int_0^\infty \int_0^\infty e^{-i(\phi_1(s)x+\phi_2(\kappa)\Upsilon)}\ e^{(\nu x+\upsilon\Upsilon)} dxd\Upsilon + \int_0^\infty \int_0^\infty e^{-i(\phi_1(s)x+\phi_2(\kappa)\Upsilon)}\ e^{-(\nu x+\upsilon\Upsilon)} dxd\Upsilon \right),$$
$$= \frac{1}{2}p_1(s)\ p_2(\kappa) \left(\int_0^\infty \int_0^\infty e^{-(i\phi_1(s)-\nu)x}\ e^{-(i\phi_2(\kappa)-\upsilon)\Upsilon} dxd\Upsilon + \int_0^\infty \int_0^\infty e^{-(i\phi_1(s)+\nu)x}\ e^{-(i\phi_2(\kappa)+\upsilon)\Upsilon} dxd\Upsilon \right),$$
$$= \frac{1}{2}\left[\frac{-p_1(s)\ p_2(\kappa)}{(i\phi_1(s)-\nu)(i\phi_2(\kappa)-\upsilon)} + \frac{p_1(s)\ p_2(\kappa)}{(i\phi_1(s)+\nu)(i\phi_2(\kappa)+\upsilon)}\right]$$
$$T_{2}{}_{g}^{c}\{cosh\ (\nu x + \upsilon\Upsilon)\ \} = \frac{-p_1(s)\ p_2(\kappa)[\phi_1(s)\phi_2(\kappa) + \nu\upsilon]}{([\phi_1(s)]^2 + \nu^2)([\phi_2(\kappa)]^2 + \upsilon^2)}.$$

Formula 5.8.
$\psi(x,\Upsilon) = (\chi\Upsilon)^\eta, \eta > 0$

$$T_{2}{}_{g}^{c}\{(\chi\Upsilon)^\eta\} = p_1(s)\ p_2(\kappa) \int_0^\infty \int_0^\infty e^{-i(\phi_1(s)x+\phi_2(\kappa)\Upsilon)}\ (x\Upsilon)^\eta dxd\Upsilon,$$
$$= p_1(s)\ p_2(\kappa) \int_0^\infty \left(\int_0^\infty e^{-i\phi_1(s)x}\ x^\eta dx\right) e^{-i\phi_2(\kappa)\Upsilon}\Upsilon^\eta d\Upsilon$$
$$= p_1(s)\ p_2(\kappa) \int_0^\infty \left[\frac{(-i)^{\eta+1}\Gamma(\eta+1)}{[\phi_1(s)]^{\eta+1}}\right] e^{-i\phi_2(\kappa)\Upsilon}\Upsilon^\eta d\Upsilon$$
$$T_{2}{}_{g}^{c}\{(x\Upsilon)^\eta\} = \frac{(-i)^{2(\eta+1)}[\Gamma(\eta+1)]^2 p_1(s)\ p_2(\kappa)}{[\phi_1(s)\phi_2(\kappa)]^{\eta+1}}.$$

Formula 5.9.
$\psi(x,\Upsilon) = x^\mu\Upsilon^\eta, \qquad \mu, \eta > 0$

$$T_{2_g}^{\ c}\{x^{\mu}\Upsilon^{\eta}\} = p_1(s)\, p_2(\kappa)\int_0^{\infty}\int_0^{\infty} e^{-i(\phi_1(s)x+\phi_2(\kappa)\Upsilon)}\, x^{\mu}\Upsilon^{\eta}dxd\Upsilon,$$

$$= p_1(s)\, p_2(\kappa)\int_0^{\infty}\left(\int_0^{\infty} e^{-i\phi_1(s)x}\, x^{\mu}dx\right)e^{-i\phi_2(\kappa)\Upsilon}\Upsilon^{\eta}d\Upsilon$$

$$T_{2_g}^{\ c}\{x^{\mu}\Upsilon^{\eta}\} = \frac{(-i)^{\mu+1}(-i)^{\eta+1}\Gamma(\mu+1)\Gamma(\eta+1)p_1(s)\, p_2(\kappa)}{[\phi_1(s)]^{\mu+1}[\phi_2(\kappa)]^{\eta+1}}. \qquad \blacksquare$$

Formula 5.10.

$\psi(x,\Upsilon) = J_0(c\sqrt{x\Upsilon})$.

$$T_{2_g}^{\ c}\{J_0(c\sqrt{x\Upsilon})\} = p_1(s)\, p_2(\kappa)\int_0^{\infty}\int_0^{\infty} e^{-i(\phi_1(s)x+\phi_2(\kappa)\Upsilon)}\, J_0(c\sqrt{x\Upsilon})dxd\Upsilon,$$

$$= p_2(\kappa)\int_0^{\infty}\left(p_1(s)\int_0^{\infty} e^{-i\phi_1(s)x}\, J_0(c\sqrt{x\Upsilon})dx\right)e^{-i\phi_2(\kappa)\Upsilon}d\Upsilon,$$

$$= p_2(\kappa)\int_0^{\infty}\left(\frac{-ip_1(s)}{\phi_1(s)}\, e^{\frac{-ic^2}{4\phi_1(s)}\Upsilon}\right)e^{-i\phi_2(\kappa)\Upsilon}d\Upsilon,$$

$$= \frac{-ip_1(s)\, p_2(\kappa)}{\phi_1(s)}\int_0^{\infty}(e^{\Upsilon})\, e^{-i\left(\phi_2(\kappa)-\frac{c^2}{4\phi_1(s)}\right)\Upsilon}d\Upsilon,$$

$$= \frac{-ip_1(s)\, p_2(\kappa)}{\phi_1(s)}\frac{1}{\left[\phi_2(\kappa)-\frac{c^2}{4\phi_1(s)}\right]}(0-1),$$

$$T_{2_g}^{\ c}\{J_0(c\sqrt{x\Upsilon})\} = \frac{-4p_1(s)\, p_2(\kappa)}{4\phi_1(s)\phi_2(\kappa)-c^2}.$$

Formula 5.11.

$\psi(x,\Upsilon) = \Omega(x)\Lambda(\Upsilon)$.

$$T_{2_g}^{\ c}\{\psi(x,\Upsilon)\} = p_1(s)\, p_2(\kappa)\int_0^{\infty}\int_0^{\infty} e^{-i(\phi_1(s)x+\phi_2(\kappa)\Upsilon)}\, \psi(x,\Upsilon)dxd\Upsilon,$$

$$= \left[p_1(s)\int_0^{\infty} e^{-i\phi_1(s)x}\, \Omega(x)dx\right]\left[p_2(\kappa)\int_0^{\infty} e^{-i\phi_2(\kappa)\Upsilon}\, \Lambda(\Upsilon)d\Upsilon\right],$$

$$T_{2_g}^{\ c}\{\psi(x,\Upsilon)\} = T_g^c\{\Omega(x)\}T_g^c\{\Lambda(\Upsilon)\}. \qquad \blacksquare$$

6. The Double SEJI Integral Transform of Partial Differential Derivatives

Here, some results related to double SEJI integral transform partial derivatives are proposed. The partial derivatives for x will be as follows:

Theorem 6.1. *Let* $T_{2_g}^{\ c}\{\psi(x,\Upsilon)\} = F_{2_g}^{\ c}(s,\kappa)$ *, then*

i. $T_{2_g}^{\ c}\left\{\frac{\partial\psi(x,\Upsilon)}{\partial x}\right\} = i\phi_1(s)F_{2_g}^{\ c}(s,\kappa) - p_1(s)T_g^c\{\psi(0,\Upsilon)\}$.

ii. $T_{2_g}^{\ c}\left\{\frac{\partial^2\psi(x,\Upsilon)}{\partial x^2}\right\} = [i\phi_1(s)]^2F_{2_g}^{\ c}(s,\kappa) - i\phi_1(s)p_1(s)T_g^c\{\psi(0,\Upsilon)\} - p_1(s)T_g^c\left\{\frac{\partial\psi}{\partial x}(0,\Upsilon)\right\}$

Proof: i.

$$T_{2_g}^{\ c}\left\{\frac{\partial\psi(x,\Upsilon)}{\partial x}\right\} = p_1(s)\, p_2(\kappa)\int_0^{\infty}\left(\int_0^{\infty} e^{-i\phi_1(s)x}\frac{\partial\psi(x,\Upsilon)}{\partial x}dx\right)e^{-i\phi_2(\kappa)\Upsilon}d\Upsilon,$$

Integrate above by parts, we get:

$$T_{2_g}^{\ c}\left\{\frac{\partial\psi(x,\Upsilon)}{\partial x}\right\} = p_1(s)\, p_2(\kappa)\int_0^{\infty}\left(e^{-i\phi_1(s)x}\int_0^{\infty}\frac{\partial\psi(x,\Upsilon)}{\partial x}dx - \int_0^{\infty}\left(-i\phi_1(s)e^{-i\phi_1(s)x}\int_0^{\infty}\frac{\partial\psi(x,\Upsilon)}{\partial x}dx\right)dx\right)e^{-i\phi_2(\kappa)\Upsilon}d\Upsilon,$$

$$= p_1(s)\, p_2(\kappa)\int_0^{\infty}\left(-\psi(0,\Upsilon) + i\phi_1(s)\int_0^{\infty} e^{-i\phi_1(s)x}\, \psi(x,\Upsilon)d\chi\right)e^{-i\phi_2(\kappa)\Upsilon}d\Upsilon,$$

$$= -p_1(s)\, p_2(\kappa)\int_0^{\infty} -\psi(0,\Upsilon)\, e^{-i\phi_2(\kappa)\Upsilon}d\Upsilon + i\phi_1(s)p_1(s)\, p_2(\kappa)\int_0^{\infty}\int_0^{\infty} e^{-i(\phi_1(s)x+\phi_2(\kappa)\Upsilon)}\, \psi(x,\Upsilon)dxd\Upsilon,$$

$$T_{2_g}^{\ c}\left\{\frac{\partial\psi(x,\Upsilon)}{\partial x}\right\} = i\phi_1(s)F_{2_g}^{\ c}(\vartheta,\kappa) - \Theta_1(\vartheta)T_g^c\{\psi(0,\Upsilon)\}. \qquad \blacksquare$$

ii. $T_{2g}^{\ c}\left\{\frac{\partial^2\psi(x,\Upsilon)}{\partial x^2}\right\} = p_1(s)\,p_2(\kappa)\int_0^\infty\left(\int_0^\infty e^{-i\phi_1(s)x}\frac{\partial^2\psi(x,\Upsilon)}{\partial x^2}dx\right)e^{-i\phi_2(\kappa)\Upsilon}d\Upsilon,$

Integrate above by parts, we get:

$T_{2g}^{\ c}\left\{\frac{\partial^2\psi(x,\Upsilon)}{\partial x^2}\right\} = p_1(s)\,p_2(\kappa)\int_0^\infty\left(e^{-i\phi_1(s)x}\int_0^\infty\frac{\partial\psi(x,\Upsilon)}{\partial x}dx - \int_0^\infty\left(-i\phi_1(s)e^{-i\phi_1(s)x}\int_0^\infty\frac{\partial^2\psi(\chi,\Upsilon)}{\partial x^2}dx\right)dx\right)e^{-i\phi_2(\kappa)\Upsilon}d\Upsilon,$

Let $u = e^{-i\Phi_1(\vartheta)x} \Rightarrow du = -i\phi_1(s)e^{-i\phi_1(s)x}dx,$

$$dv = \frac{\partial^2\psi(x,\Upsilon)}{\partial x^2}d\chi \Rightarrow v = \int_0^\infty\frac{\partial^3\psi(x,\Upsilon)}{\partial x^3}dx.$$

Then:

$$T_{2g}^{\ c}\left\{\frac{\partial^2\psi(x,\Upsilon)}{\partial x^2}\right\} = p_2(\kappa)\int_0^\infty e^{-i\phi_2(\kappa)\Upsilon}\,d\Upsilon\left[p_1(s)\left(e^{-i\phi_1(s)x}\int_0^\infty\left(\frac{\partial^2\psi(x,\Upsilon)}{\partial x^2}\right)dx\right.\right.$$
$$\left.\left.+\,i\phi_1(s)\int_0^\infty e^{-i\phi_1(s)x}\left(\int_0^\infty\frac{\partial^2\psi(x,y)}{\partial x^2}dx\right)dx\right)\right],$$
$$= p_1(s)\,p_2(\kappa)\int_0^\infty e^{-i\phi_2(\kappa)\Upsilon}\left(\frac{-\partial\psi(0,\Upsilon)}{\partial x} + i\phi_1(s)\int_0^\infty e^{-i\phi_1(s)x}\frac{\partial\psi(x,\Upsilon)}{\partial x}dx\right)d\Upsilon,$$
$$= p_1(s)\,p_2(\kappa)\int_0^\infty e^{-i\phi_2(\kappa)\Upsilon}\frac{\partial^2\psi(0,\Upsilon)}{\partial x^2}d\Upsilon + i\phi_1(s)p_1(s)\,p_2(\kappa)\int_0^\infty\int_0^\infty e^{-i(\phi_1(s)x+\phi_2(\kappa)\Upsilon)}\left(\frac{\partial\psi(x,\Upsilon)}{\partial x}\right)dxd\Upsilon,$$
$$T_{2g}^{\ c}\left\{\frac{\partial^2\psi(x,\Upsilon)}{\partial x^2}\right\} = -p_1(s)T_g^c\left\{\frac{\partial\psi(0,\Upsilon)}{\partial x}\right\} + i\phi_1(s)T_{2g}^{\ c}\left\{\frac{\partial\psi(x,\Upsilon)}{\partial x}\right\}.$$

By substituting $T_{2g}^{\ c}\left\{\frac{\partial\psi(x,\Upsilon)}{\partial x}\right\}$ we get:

$$T_{2g}^{\ c}\left\{\frac{\partial^2\psi(x,\Upsilon)}{\partial x^2}\right\} = [i\phi_1(s)]^2F_{2g}^{\ c}(\vartheta,\kappa) - i\phi_1(s)p_1(s)T_g^c\{\psi(0,\Upsilon)\} - p_1(s)T_g^c\left\{\frac{\partial\psi}{\partial x}(0,\Upsilon)\right\}. \quad \blacksquare$$

In general,

$$T_{2g}^{\ c}\left\{\frac{\partial^n\psi(x,\Upsilon)}{\partial x^n}\right\} = \left(i\phi_1(s)\right)^nF_{2g}^{\ c}(s,\kappa) - p_1(s)\left[\sum_{k=0}^{n-1}\left(i\phi_1(s)\right)^{n-k-1}T_g^c\left\{\frac{\partial^k\psi}{\partial x^k}(0,\Upsilon)\right\}\right].$$

We can prove the above formula By Mathematical Induction,
for $n = 1$, we proved it later.
Suppose that true for $n = m$ that means:

$$T_{2g}^{\ c}\left\{\frac{\partial^m\psi(x,\Upsilon)}{\partial x^m}\right\} = \left(i\phi_1(s)\right)^mF_{2g}^{\ c}(s,\kappa) - p_1(s)\left[\sum_{k=0}^{m-1}\left(i\phi_1(s)\right)^{m-k-1}T_g^c\left\{\frac{\partial^k\psi}{\partial x^k}(0,\Upsilon)\right\}\right],$$

We want to prove that $n = m + 1$

$$T_{2g}^{\ c}\left\{\frac{\partial^{m+1}\psi(x,\Upsilon)}{\partial x^{m+1}}\right\} = T_g^c\left\{\frac{\partial}{\partial x}\left(\frac{\partial^m\psi(x,\Upsilon)}{\partial x^m}\right)\right\} = \left(i\phi_1(s)\right)T_g^c\left\{\left(\frac{\partial^m\psi(x,\Upsilon)}{\partial x^m}\right)\right\} - p_1(s)T_g^c\left\{\frac{\partial^m\psi}{\partial x^m}(0,\Upsilon)\right\},$$
$$= \left(i\phi_1(s)\right)\left(\left(i\phi_1(s)\right)^mF_{2g}^{\ c}(s,\kappa) - p_1(s)\left[\sum_{k=0}^{m-1}\left(i\phi_1(s)\right)^{m-k-1}T_g^c\left\{\frac{\partial^k\psi}{\partial x^k}(0,\Upsilon)\right\}\right]\right) - p_1(s)T_g^c\left\{\frac{\partial^m\psi}{\partial x^m}(0,\Upsilon)\right\},$$
$$= \left(i\phi_1(s)\right)^{m+1}F_{2g}^{\ c}(s,\kappa) - p_1(s)\sum_{k=0}^{m-1}\left(i\phi_1(s)\right)^{m-k-1}T_g^c\left\{\frac{\partial^k\psi}{\partial x^k}(0,\Upsilon)\right\} - p_1(s)T_g^c\left\{\frac{\partial^m\psi}{\partial x^m}(0,\Upsilon)\right\},$$
$$= \left(i\phi_1(s)\right)^{m+1}F_{2g}^{\ c}(s,\kappa) - p_1(s)\left[\sum_{k=0}^{m-1}\left(i\phi_1(s)\right)^{m-k-1}T_g^c\left\{\frac{\partial^k\psi}{\partial x^k}(0,\Upsilon)\right\} + T_g^c\left\{\frac{\partial^m\psi}{\partial x^m}(0,\Upsilon)\right\}\right],$$
$$= \left(i\phi_1(s)\right)^{m+1}F_{2g}^{\ c}(s,\kappa) - p_1(s)\left[\sum_{k=0}^{m}\left(i\phi_1(s)\right)^{m-k-1}T_g^c\left\{\frac{\partial^k\psi}{\partial x^k}(0,\Upsilon)\right\}\right],$$
$$= T_{2g}^{\ c}\left\{\frac{\partial^{m+1}\psi(x,\Upsilon)}{\partial x^{m+1}}\right\}.$$

So the theorem is true for $n \in N$. $\blacksquare$

Theorem 6.2. *Let* $T_{2g}^{\ c}\{\psi(x,\Upsilon)\} = F_{2g}^{\ c}(s,r)$ *, then*

i. $T_{2g}^{\ c}\left\{\frac{\partial\psi(x,\Upsilon)}{\partial\Upsilon}\right\} = i\phi_2(\kappa)F_{2g}^{\ c}(s,\kappa) - p_2(\kappa)T_g^c\{\psi(x,0)\}.$

ii. $T_{2g}^{\ c}\left\{\frac{\partial^2\psi(x,\Upsilon)}{\partial\Upsilon^2}\right\} = [i\phi_2(\kappa)]^2F_{2g}^{\ c}(s,\kappa) - i\phi_2(\kappa)\,p_2(\kappa)T_g^c\{\psi(x,0)\} - p_2(\kappa)T_g^c\left\{\frac{\partial\psi}{\partial\Upsilon}(x,0)\right\}.$

iii. $T_{2g}^{\ c}\left\{\frac{\partial^n\psi(x,\Upsilon)}{\partial\Upsilon^n}\right\} = \left(i\phi_2(\kappa)\right)^nF_{2g}^{\ c}(s,\kappa) - p_2(\kappa)\left[\sum_{k=0}^{n-1}\left(i\phi_2(\kappa)\right)^{n-k-1}T_g^c\left\{\frac{\partial^k\psi}{\partial\Upsilon^k}(x,0)\right\}\right].$

This theorem has the same approach of proof but with respect to γ.

Theorem 6.3. *The double SEJI integral transform of the partial derivative for x, Υ is constructed by:*

$T_{2_g}^{c}\left\{\frac{\partial^2\psi(x,\Upsilon)}{\partial x\partial y}\right\} = i^2 p_1(s)\, p_2(\kappa) F_{2_g}^{c}(s,\kappa) - i\phi_2(\kappa)p_1(s)T_g^c\{\psi(0,\Upsilon)\} - i\phi_1(s)p_2(\kappa)T_g^c\{\psi(x,0)\} - p_1(s)p_2(\kappa)\psi(0,0).$

Proof: we have

$$T_{2_g}^{c}\left\{\frac{\partial^2\psi(x,\Upsilon)}{\partial x\partial \Upsilon}\right\} = p_1(s)\, p_2(\kappa)\int_0^\infty \int_0^\infty e^{-i(\phi_1(s)x+\phi_2(\kappa)\Upsilon)}\frac{\partial^2\psi(x,\Upsilon)}{\partial x\partial \Upsilon}dxd\Upsilon,$$

$$T_{2_g}^{c}\left\{\frac{\partial^2\psi(\chi,\Upsilon)}{\partial \chi\partial \Upsilon}\right\} = p_1(s)\int_0^\infty e^{-i\phi_1(s)x}\left(p_2(\kappa)\int_0^\infty e^{-i\phi_2(\kappa)\Upsilon}\frac{\partial^2\psi(x,\Upsilon)}{\partial x^2}dx\right)d\Upsilon,$$

Integrate above by parts with respect to γ, we get:

$$T_{2_g}^{c}\left\{\frac{\partial^2\psi(x,\Upsilon)}{\partial x\partial \Upsilon}\right\} =$$

$$-p_1(s)\, p_2(\kappa)\int_0^\infty e^{-i\phi_1(s)x}\frac{\partial\psi(x,0)}{\partial x}dx + i\phi_2(\kappa)p_1(s)\, p_2(\kappa)\int_0^\infty\int_0^\infty \frac{\partial\psi(x,\Upsilon)}{\partial x}e^{-i(\phi_1(s)x+\phi_2(\kappa)\Upsilon)}dxd\Upsilon,$$

$$= -\, p_2(\kappa)T_g^c\left\{\frac{\partial\psi(x,0)}{\partial x}\right\} + i\phi_2(\kappa)T_{2_g}^{c}\left\{\frac{\partial\psi(x,\Upsilon)}{\partial x}\right\},$$

$$= -\, p_2(\kappa)\left[i\phi_1(s)T_g^c\{\psi(x,0)\} - p_1(s)\psi(0,0)\right] + i\phi_2(\kappa)\left[i\phi_1(s)T_{2_g}^{c}\{\psi(x,\Upsilon)\} - p_1(s)T_g^c\{\psi(0,\Upsilon)\}\right],$$

$$T_{2_g}^{c}\left\{\frac{\partial^2\psi(x,\Upsilon)}{\partial x\partial \Upsilon}\right\} = i^2 p_1(s)\, p_2(\kappa)F_{2_g}^{c}(\vartheta,\kappa) - i\phi_2(\kappa)p_1(s)T_g^c\{\psi(0,\Upsilon)\} - i\phi_1(s)\, p_2(\kappa)T_g^c\{\psi(x,0)\} - p_1(s)p_2(\kappa)\psi(0,0). \quad \blacksquare$$

Corollary 6.1. *Let* $T_{2_g}^{c}\{\psi(x,\Upsilon)\} = F_{2_g}^{c}(s,\kappa)$ *, then*

$$T_{2_g}^{c}\left\{\int_0^x\int_0^\Upsilon \psi(\nu,\upsilon)\, d\nu d\upsilon\right\} = \frac{-1}{\phi_1(s)\phi_2(\kappa)}F_{2_g}^{c}(s,\kappa),$$

Where $\phi_1(s), \phi_2(\kappa) \neq 0,\ \ \forall s,\kappa \in R^+$.

then

Proof:

Consider $T_{2_g}^{c}\{\Omega(x,\Upsilon)\}$ to be the SEJI double integral transform of the function $h(x,\Upsilon)$ as identified by

$$\Omega(x,y) = \int_0^x\int_0^\Upsilon \psi(\nu,\upsilon)\, d\nu d\upsilon.$$

Clearly, we have $h_{x\Upsilon}(x,\Upsilon) = f(x,\Upsilon)$ and $\Omega(0,0) = 0$. Therefore,

$$T_{2_g}^{c}\{\Omega_{x\Upsilon}(x,\Upsilon)\} = T_{2_g}^{c}\{\psi(x,\Upsilon)\} = F_{2_g}^{c}(s,\kappa).$$

By the theorem (3) , we obtain

$$F_{2_g}^{c}(s,\kappa) = -\phi_1(s)\phi_2(\kappa)T_{2_g}^{c}\{\Omega(\chi,\Upsilon)\} - i\phi_2(\kappa)p_1(s)T_g^c\{\Omega(0,\Upsilon)\} - i\phi_1(s)\, p_2(\kappa)T_g^c\{\Omega(x,0)\} - p_1(s)\, p_2(\kappa)\Omega(0,0).$$

Thus,

$$T_{2_g}^{c}\{\Omega(x,\Upsilon)\} = \frac{-1}{\phi_1(s)\phi_2(\kappa)}F_{2_g}^{c}(s,\kappa) - i\frac{p_1(s)}{\phi_1(s)}T_g^c\{\Omega(0,\Upsilon)\} - \frac{i\, p_2(\kappa)}{\phi_2(\kappa)}T_g^c\{\Omega(x,0)\}.$$

We have$T_g^c\{\Omega(x,0)\} = T_g^c\{\Omega(0,\Upsilon)\} = 0$. Then

$$T_{2_g}^{c}\{\Omega(x,\Upsilon)\} = \frac{-1}{\phi_1(s)\phi_2(\kappa)}F_{2_g}^{c}(s,\kappa). \quad \blacksquare$$

Theorem 6.4. *Let* $T_{2_g}^{c}\{\psi(x,\Upsilon)\} = F_{2_g}^{c}(s,\kappa)$ *, then*

$$T_{2_g}^{c}\{\psi(x-\varrho,\Upsilon-\sigma)H(x-\varrho,\Upsilon-\sigma)\} = e^{-i(\phi_1(s)\varrho+\phi_2(\kappa)\sigma)}F_{2_g}^{c}(s,\kappa),$$

where $H(x,\Upsilon)$ is the Heaviside unit step function defined by

$$H(x-\varrho,\Upsilon-\sigma) = \{1, \quad x>\varrho, \Upsilon>\sigma \ \ 0 \qquad otherwise.$$

Proof:

$$T_{2_g}^{c}\{\psi(x-\varrho,\Upsilon-\sigma)H(x-\varrho,\Upsilon-\sigma)\}$$

$$= p_1(s)p_2(\kappa)\int_0^\infty\int_0^\infty e^{-i(\phi_1(s)x+\phi_2(\kappa)\Upsilon)}\,[\psi(x-\varrho,y-\sigma)H(x-\varrho,\Upsilon-\sigma)]dxd\Upsilon,$$

$$= p_1(s)p_2(\kappa)\int_0^\infty\int_0^\infty e^{-i(\phi_1(s)x+\phi_2(\kappa)\Upsilon)}\,\psi(x-\varrho,\Upsilon-\sigma)dxd\Upsilon,$$

Put $w = x-\varrho, z = \Upsilon-\sigma,$

$$= p_1(s)p_2(\kappa)\int_0^\infty \int_0^\infty e^{-i\phi_1(s)(w+\varrho)-i\phi_2(\kappa)(z+\sigma)}\, \psi(w,z)dwdz,$$

$$T_{2g}^{\,c}\{\psi(x-\varrho,\Upsilon-\sigma)H(x-\varrho,\Upsilon-\sigma)\} = e^{-i(\phi_1(s)\varrho+\phi_2(\kappa)\sigma)}F_{2g}^{\,c}(s,\kappa). \qquad \blacksquare$$

Theorem 6.5. *Let* $T_{2g}^{\,c}\{\psi(x,\Upsilon)\} = F_{2g}^{\,c}(s,\kappa)$ *, then the double SEJI integral transform for the partial fractional Caputo derivatives [28] are:*

i. $T_{2g}^{\,c}\left\{\frac{\partial^{\gamma_1}\psi(x,\gamma)}{\partial x^{\gamma_1}}\right\} = (i\phi_1(s))^{\gamma_1}F_{2g}^{\,c}(s,\kappa) - p_1(s)\left[\sum_{k=0}^{m-1}(i\phi_1(s))^{\gamma_1-k-1}T_g^c\left\{\frac{\partial^k\psi(0,\Upsilon)}{\partial x^k}\right\}\right]$, $m-1<\gamma\le m,\ \ m\in\mathbb{N}$

ii. $T_{2g}^{\,c}\left\{\frac{\partial^{\gamma_2}\psi(x,\gamma)}{\partial \gamma^{\gamma_2}}\right\} = (i\phi_2(\kappa))^{\gamma_2}F_{2g}^{\,c}(s,\kappa) - p_2(\kappa)\left[\sum_{k=0}^{n-1}(i\phi_2(\kappa))^{\gamma_2-k-1}T_g^c\left\{\frac{\partial^k\psi(x,0)}{\partial \gamma^k}\right\}\right]$, $1<\gamma\le n, n\in\mathbb{N}$.

This theorem has the same approach of theorem (6.1) but with fractional orders.

7. Applications

7.1. Volterra Integral Equation:

The linear Volterra Integral Equation is thought to have

$$\psi(\chi,\Upsilon) = \zeta(x,\Upsilon) + \lambda\int_0^x\int_0^\Upsilon \psi(x-\varrho,\Upsilon-\sigma)\,\rho(\varrho,\sigma)d\varrho d\sigma, \tag{3}$$

where $\psi(x,\Upsilon)$ is the unknown function, $\lambda\in R$ and $\zeta(x,\Upsilon),\rho(x,\Upsilon)$ are two well-known functions. Using the double SEJI integral transform for eq. (3) and theorem (4.1) to get

$$F_{2g}^{\,c}(s,\kappa) = T_{2g}^{\,c}\{\zeta(x,\Upsilon)\} + \frac{\lambda}{p_1(s)p_2(\kappa)}F_{2g}^{\,c}(s,\kappa)T_{2g}^{\,c}\{\rho(x,\Upsilon)\},$$

$$F_{2g}^{\,c}(s,\kappa)\left[1-\frac{\lambda}{p_1(s)p_2(\kappa)}T_{2g}^{\,c}\{\rho(x,\Upsilon)\}\right] = T_{2g}^{\,c}\{\zeta(x,\Upsilon)\},$$

$$F_{2g}^{\,c}(s,\kappa) = \frac{T_{2g}^{\,c}\{\zeta(x,y)\}}{\left[1-\frac{\lambda}{p_1(s)p_2(\kappa)}T_{2g}^{\,c}\{\rho(x,\Upsilon)\}\right]}. \tag{4}$$

By the inverse of this transform, the solution of eq. (3) is getting.

$$\psi(\chi,\Upsilon) = T_{2g}^{\,c\,-1}\left\{\frac{T_{2g}^{\,c}\{\zeta(x,y)\}}{\left[1-\frac{\lambda}{p_1(s)p_2(\kappa)}T_{2g}^{\,c}\{\rho(x,\Upsilon)\}\right]}\right\}.$$

We can see obviously by the following example

Example 1

To solve the following equation

$$\psi(x,\Upsilon) = \delta - \lambda\int_0^x\int_0^\Upsilon \psi(\varrho,\sigma)\, d\varrho d\sigma, \tag{5}$$

Where $\delta,\lambda\in R$. [15]

Applying double SEJI integral transform of eq. (5) and by (4), we get

$$F_{2g}^{\,c}(s,\kappa) = \delta\frac{(-i)^2p_1(s)p_2(\kappa)}{\phi_1(s)\phi_2(\kappa)} - \frac{\lambda}{p_1(s)p_2(\kappa)}\cdot\frac{(-i)^2p_1(s)p_2(\kappa)}{\phi_1(s)\phi_2(\kappa)}F_{2g}^{\,c}(\vartheta,\kappa),$$

$$\Rightarrow F_{2g}^{\,c}(s,\kappa) = \frac{\delta\frac{-p_1(s)p_2(\kappa)}{\phi_1(s)\phi_2(\kappa)}}{1-\frac{\lambda}{\phi_1(s)\phi_2(\kappa)}} = \frac{-\delta p_1(s)p_2(\kappa)}{\phi_1(s)\phi_2(\kappa)-\lambda},$$

By taking the inverse of this transform we obtained the solution of $\psi(\chi,\Upsilon)$ in eq. (5)

$$\psi(\chi,\Upsilon) = T_{2g}^{\,c\,-1}\left\{\frac{-\delta p_1(s)p_2(\kappa)}{\phi_1(s)\phi_2(\kappa)-\lambda}\right\},$$

$$= \delta T_{2g}^{\,c\,-1}\left\{\frac{-p_1(s)p_2(\kappa)}{\phi_1(s)\phi_2(\kappa)-\lambda}\cdot\frac{4}{4}\right\} = \delta T_{2g}^{\,c\,-1}\left\{\frac{-4p_1(s)p_2(\kappa)}{4\phi_1(s)\phi_2(\kappa)-4\lambda}\right\},$$

$$\psi(\chi,\Upsilon)=\delta J_0\left(2\sqrt{\lambda x\Upsilon}\right).$$

Example 2:

For solving the equation

$$a^2\Upsilon=\int_0^x\int_0^y\psi(x-\varrho,y-\sigma)\Phi(\varrho,\sigma)\,d\varrho\,d\sigma, \tag{6}$$

Where $a\in\mathbb{R}$.

Applying double SEJI integral transform of eq. (6) and by eq. (4), we get

$$a^2T_{2_g}^{\,c}\{\Upsilon\}=\frac{1}{p_1(s)p_2(\kappa)}\cdot\left[F_{2_g}^{\,c}(s,\kappa)\right]^2,$$

$$a^2\frac{(-i)^3p_1(s)p_2(\kappa)}{\phi_1(s)[\phi_2(\kappa)]^2}=\frac{1}{p_1(s)p_2(\kappa)}\cdot\left[F_{2_g}^{\,c}(s,\kappa)\right]^2,$$

$$\left[F_{2_g}^{\,c}(s,\kappa)\right]^2=\frac{a^2(-i)^3[p_1(s)p_2(\kappa)]^2}{\phi_1(s)[\phi_2(\kappa)]^2},$$

$$F_{2_g}^{\,c}(s,\kappa)=\frac{a(-i)^{\frac{3}{2}}p_1(s)p_2(\kappa)}{\left[\phi_1(s)\right]^{\frac{1}{2}}\phi_2(\kappa)},$$

By taking the inverse of this transform we obtained the solution of $f(x,y)$ in eq. (6).

$$\psi(x,\Upsilon)=aT_{2_g}^{\,c\,-1}\left\{\frac{(-i)^{\frac{3}{2}}p_1(s)p_2(\kappa)}{[\phi_1(s)]^{\frac{1}{2}}\phi_2(\kappa)}\right\},$$

$$=aT_{2_g}^{\,c\,-1}\left\{\frac{(-i)^{\frac{3}{2}}p_1(s)p_2(\kappa)}{[\phi_1(s)]^{\frac{1}{2}}\phi_2(\kappa)}\cdot\frac{\sqrt{\pi}}{\sqrt{\pi}}\right\},$$

$$\psi(\chi,\Upsilon)=\frac{a}{\sqrt{\pi}}\frac{1}{\sqrt{x}}.$$

7.2. Volterra Integro- partial differential Equations:

The linear Volterra Integro- partial differential equation is represented by, [15]:

$$\frac{\partial\psi(x,\Upsilon)}{\partial\chi}+\frac{\partial\psi(x,\Upsilon)}{\partial\Upsilon}=\zeta(\chi,\Upsilon)+\lambda\int_0^x\int_0^\Upsilon\psi(x-\varrho,\Upsilon-\sigma)\,\rho(\varrho,\sigma)d\varrho d\sigma, \tag{7}$$

with the conditions:

$$\psi(x,0)=\psi_0(x),\qquad \psi(0,\Upsilon)=h_0(\Upsilon).$$

where $\psi(\chi,\Upsilon)$ is the unknown function, $\lambda\in R$ and $\zeta(x,\Upsilon),\rho(x,\Upsilon)$ are two known functions.

By effecting the double SEJI integral transform for eq. (7) and using the single SEJI transform for the conditions, we get

$$i\phi_1(s)F_{2_g}^{\,c}(s,\kappa)-p_1(s)T_g^c\{h_0(\Upsilon)\}+i\phi_2(\kappa)F_{2_g}^{\,c}(s,\kappa)-p_2(\kappa)T_g^c\{\psi_0(x)\}$$
$$=T_{2_g}^{\,c}\{\zeta(x,\Upsilon)\}+\frac{\lambda}{p_1(s)p_2(\kappa)}F_{2_g}^{\,c}(s,\kappa)T_{2_g}^{\,c}\{\rho(x,\Upsilon)\},$$

$$F_{2_g}^{\,c}(s,\kappa)=\frac{T_{2_g}^{\,c}\{\zeta(x,\Upsilon)\}+p_1(s)T_g^c\{h_0(\Upsilon)\}+p_2(\kappa)T_g^c\{\psi_0(x)\}}{\left[i\phi_1(s)+i\phi_2(\kappa)-\frac{\lambda}{p_1(s)p_2(\kappa)}T_{2_g}^{\,c}\{\rho(x,\Upsilon)\}\right]}. \tag{8}$$

Then the solution of eq. (7)

$$\psi(x,\Upsilon) = {T_2}_g^{c^{-1}}\left\{\frac{{T_2}_g^c\{\zeta(x,\Upsilon)\} + p_1(s)T_g^c\{h_0(\Upsilon)\} + p_2(\kappa)T_g^c\{\psi_0(x)\}}{\left[i\phi_1(s) + i\phi_2(\kappa) - \frac{\lambda}{p_1(s)p_2(\kappa)}{T_2}_g^c\{\rho(x,\Upsilon)\}\right]}\right\}.$$

We can see obviously by the following example

Example3:

To solve the following equation

$$\frac{\partial\psi(x,\Upsilon)}{\partial x} + \frac{\partial\psi(x,\Upsilon)}{\partial\Upsilon} = -1 + e^x + e^\Upsilon + e^{x+\Upsilon} + \int_0^x\int_0^\Upsilon \psi(x-\varrho,\Upsilon-\sigma)\,d\varrho d\sigma, \tag{9}$$

with the conditions:

$$\psi(x,0) = \psi_0(x) = e^x, \qquad \psi(0,\Upsilon) = h_0(y) = e^\Upsilon.$$

where $\lambda = 1, \rho(\varrho,\sigma) = 1$. [15]

Applying double SEJI integral transform of eq. (9) and using the single SEJI transform for the conditions, we get

$$F_{2_g}^c(s,\kappa) = \frac{\frac{p_1(s)p_2(\kappa)[i+2i\phi_1(s)\phi_2(\kappa)]}{(i\phi_1(s)-1)(i\phi_2(\kappa)-1)i\phi_1(s)\phi_2(\kappa)} - \frac{p_1(s)p_2(\kappa)[1+\phi_2(\kappa)]}{1+[\phi_2(\kappa)]^2} - \frac{p_1(s)p_2(\kappa)[1+\phi_1(s)]}{1+[\phi_1(s)]^2}}{i\phi_1(s) + i\phi_2(\kappa) - \frac{1}{p_1(s)p_2(\kappa)}\cdot\frac{-p_1(s)p_2(\kappa)}{\phi_1(s)\phi_2(\kappa)}},$$

$$F_{2_g}^c(s,r) = \frac{p_1(s)p_2(\kappa)}{(i\phi_1(s)-1)(i\phi_2(\kappa)-1)},$$

By taking the inverse of this transform we obtained the solution of $\psi(x,\Upsilon)$ in eq. (9)

$$\psi(x,\Upsilon) = {T_2}_g^{c^{-1}}\left\{\frac{p_1(s)p_2(\kappa)}{(i\phi_1(s)-1)(i\phi_2(\kappa)-1)}\right\} = e^{x+\Upsilon}.$$

7.2. Partial Integro- differential Equations:

The form of the linear partial Integro-differential equation is as follows:

$$\frac{\partial^2\psi(x,\Upsilon)}{\partial\Upsilon^2} - \frac{\partial^2\psi(x,\Upsilon)}{\partial x^2} + \psi(x,\Upsilon) + \int_0^x\int_0^\Upsilon \psi(x-\varrho,\Upsilon-\sigma)\,\rho(\varrho,\sigma)d\varrho d\sigma = \zeta(x,\Upsilon), \tag{10}$$

with the conditions:

$$\psi(x,0) = \psi_0(x), \frac{\partial\psi(x,0)}{\partial\Upsilon} = \psi_1(x),\ \ \psi(0,\Upsilon) = h_0(\Upsilon), \frac{\partial\psi(0,\Upsilon)}{\partial x} = h_1(x).$$ [15]

$$[i\phi_2(\kappa)]^2F_{2_g}^c(s,\kappa) - i\phi_2(\kappa)p_2(\kappa)T_g^c\{\psi_0(x)\} - p_2(\kappa)T_g^c\left\{\frac{\partial\psi(x,0)}{\partial\Upsilon}\right\} - [i\phi_1(s)]^2F_{2_g}^c(s,\kappa) + i\phi_1(s)p_1(s)T_g^c\{h_0(\Upsilon)\} + p_1(s)T_g^c\left\{\frac{\partial\psi(0,\Upsilon)}{\partial x}\right\} + F_{2_g}^c(s,\kappa) + \frac{1}{p_1(s)p_2(\kappa)}F_{2_g}^c(\vartheta,\kappa){T_2}_g^c\{\rho(x,\Upsilon)\} = {T_2}_g^c\{\zeta(x,\Upsilon)\},$$

$$F_{2_g}^c(s,\kappa) = \frac{{T_2}_g^c\{\zeta(x,\Upsilon)\} + i\phi_2(\kappa)p_2(\kappa)T_g^c\{\psi_0(x)\} + p_2(\kappa)T_g^c\left\{\frac{\partial\psi(x,0)}{\partial\Upsilon}\right\} - i\phi_1(s)p_1(s)T_g^c\{h_0(\Upsilon)\} - p_1(s)T_g^c\left\{\frac{\partial\psi(0,\Upsilon)}{\partial x}\right\}}{[i\phi_2(\kappa)]^2 - [i\phi_1(s)]^2 + 1 + \frac{1}{p_1(s)p_2(\kappa)}{T_2}_g^c\{\rho(x,\Upsilon)\}}. \tag{11}$$

To solve eq. (10), the inverse of the double SEJI integral transform is applied.

$$\psi(x,\Upsilon) = {T_2}_g^{c^{-1}}\left\{\frac{{T_2}_g^c\{\zeta(x,\Upsilon)\} + i\phi_2(\kappa)p_2(\kappa)T_g^c\{\psi_0(x)\} + p_2(\kappa)T_g^c\left\{\frac{\partial\psi(x,0)}{\partial\Upsilon}\right\} - i\phi_1(s)p_1(s)T_g^c\{h_0(\Upsilon)\} - p_1(s)T_g^c\left\{\frac{\partial\psi(0,\Upsilon)}{\partial\chi}\right\}}{[i\phi_2(\kappa)]^2 - [i\phi_1(s)]^2 + 1 + \frac{1}{p_1(s)p_2(\kappa)}{T_2}_g^c\{\rho(x,\Upsilon)\}}\right\}.$$

We can see obviously by the following example

Example4:

To solve the following equation

$$\frac{\partial^2\psi(x,\Upsilon)}{\partial\Upsilon^2} - \frac{\partial^2\psi(x,\Upsilon)}{\partial x^2} + \psi(x,\Upsilon) + \int_0^x\int_0^\Upsilon e^{x-\varrho+\Upsilon-\sigma}\,\rho(\varrho,\sigma)d\varrho d\sigma = e^{x+\Upsilon} + x\Upsilon e^{x+\Upsilon} \tag{12}$$

with the conditions:

$$\psi(x,0)=\psi_0(x)=e^x,\frac{\partial\psi(x,0)}{\partial\Upsilon}=\psi_1(x)=e^x,\ \ \psi(0,\Upsilon)=h_0(\Upsilon)=e^\Upsilon,\frac{\partial\psi(0,\Upsilon)}{\partial x}=h_1(\Upsilon)=e^\Upsilon.[15]$$

Applying double SEJI integral transform of eq. (12) and using the single SEJI transform for the conditions and simplification, we get

$$F_{2_g}^{\ c}(s,\kappa)=\frac{\frac{p_1(s)p_2(\kappa)[(i\phi_1(s)-1)(i\phi_2(\kappa)-1)([i\phi_2(\kappa)]^2-[\phi_1(s)]^2+1)+1]}{(i\phi_1(s)-1)(i\phi_2(\kappa)-1)}}{(i\phi_1(s)-1)(i\phi_2(\kappa)-1)([i\phi_2(\kappa)]^2-[i\phi_1(s)]^2+1)+1},$$

$$F_{2_g}^{\ c}(s,\kappa)=\frac{p_1(s)p_2(\kappa)}{(i\phi_1(s)-1)(i\phi_2(\kappa)-1)},$$

By taking the inverse of this transform we obtained the solution of $\psi(x,y)$ in eq. (12).

$$\psi(x,y)=T_{2_g}^{\ c^{-1}}\left\{\frac{p_1(s)p_2(\kappa)}{(i\phi_1(s)-1)(i\phi_2(\kappa)-1)}\right\}=e^{x+\Upsilon}.$$

Conclusion

In this paper, the double SEJI integral transform is depended for solving some types of integral differential equations. We can introduce important theorems and main properties for this new transform. The solved applications in this work show that the suggested transform is almost perfect in solving integral differential equations. In future, we can use this transform and solve kinds of fractional integral equations.

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