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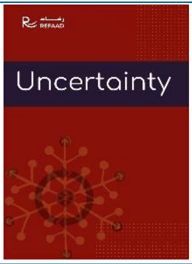

Intuitionistic Possibility Shadow Soft Sets Theory and its Applications

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Abstract

Shadow soft set is a new concept defined as a new tool with uncertainty where the values of membership taken from 0, 1 and [0,1]. In this thesis as a generalization of shadow soft set we introduce a new concept which is an Intuitionistic possibility shadow soft set and study its properties. Furthermore, some examples of Intuitionistic possibility shadow soft set and its properties are presented. We also introduce some of the operations of this concept and give some results related to these operations. Finally, the definitions of (AND) and (OR) operations are given and the properties of these operations in their application to decision-making problems are shown.

Keywords: Fuzzy set, Intuitionistic Fuzzy Set, soft set, Shadow set, Fuzzy soft set, Possibility Fuzzy soft set, shadow soft set, Possibility Shadow soft set, Intuitionistic possibility Fuzzy soft set.

2010 MSC: MSC code1, MSC code2, more.

1 Introduction

In most real-life problems in social sciences, engineering, medical sciences, and economics the data involved are imprecise in nature. The solutions to such problems involve the use of mathematical principles based on uncertainty and imprecision. A number of theories have been proposed for dealing with uncertainties in an efficient way. The fuzzy set was introduced by Zadeh [1] as a mathematical way to represent and deal with vagueness in everyday life. Then Atanassov defined the concept of the intuitionistic fuzzy set which is more general than a fuzzy set [2]. Pedrycz introduced the concept of shadowed sets by using the concept of a fuzzy set [3]. Molodtsov initiated the theory of soft sets as a new mathematical tool for dealing with uncertainties that traditional mathematical tools cannot handle [4]. Maji have further studied the theory of soft sets and used this theory to solve some decision-making problems [5], [6]. Baesho developed the idea of generalized intuitionistic fuzzy soft sets, where the definition of an intuitionistic fuzzy soft set attaches a degree to the parameterization of fuzzy sets [7]. Alkhazaleh and Salleh defined the concept of a soft expert set and they gave an application of this concept to decision-making [8]. Also, Maji introduced the concept of fuzzy soft set and studied its properties [9].

Roy and Maji used this theory to solve some decision-making problems [10]. Alkhazaleh defined the concept of fuzzy parameterized interval-valued fuzzy soft set and gave its applications in decision-making and

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medical diagnosis and defined the concept of possibility fuzzy soft set and gave its applications in decision-making and medical diagnosis [8]. Maji defined the concept of intuitionistic fuzzy soft set [11]. Salleh gave a brief survey from soft set to intuitionistic fuzzy soft set [12]. In this paper, we generalize the concept of possibility shadow soft set to the Intuitionistic possibility shadow soft set. Bashir generalizes the concept of the possibility intuitionistic fuzzy soft set and also give some applications of the possibility intuitionistic fuzzy soft set in decision-making problems and medical diagnosis [13]. In 2022, Alkhazaleh introduced a new concept by developing the concept of a Fuzzy Soft set, in which the shadow soft set focuses on the membership function. Alkhazaleh, S introduces the concept of a shadow soft set as a combination of the shadow set and soft set [14]. Alzghoul introduced the concept of the possibility shadow soft set and studied its properties [15].

In our generalization, a possibility of each element in the universe is attached to the parameterization of shadow sets while defining an Intuitionistic possibility shadow soft set. We also give some applications of the Intuitionistic possibility shadow soft set in decision-making problems and medical diagnosis.

2 Preliminaries

Preliminaries, in this section we recall some definitions and properties regarding intuitionistic fuzzy soft set and possibility shadow soft set required in this paper.

Definition 2.1. *(Fuzzy set) [1]: Let U be a Universe Set. Let $A \in U$ and U be characterized by a member function $\mu_A(x)$ that takes two values in the interval [0,1], the fuzzy set*

$$A = \{ x , \mu_A(x) , \forall x \in U \} \tag{1}$$

Definition 2.2. *(Intuitionistic fuzzy set) [16]): An intuitionistic fuzzy set (IFS) A in a nonempty set U (a universe of discourse) is an object having the form $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in U\}$, where the functions*

$$\mu_A(x): U \to [0,1], \nu_A(x): U \to [0,1] \tag{2}$$

denotes the degree of membership and degree of nonmembership of each element $x \in U$ to the set A, respectively, and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for all $x \in U$.

Definition 2.3. *(Shadow set) [3]): Let U be a set of objects, called the universe. A shadowed set on U is any mapping $s : U \longrightarrow \{0,1,[0,1]\}$. We denote the collection of all shadowed sets on U as $\{0,1,[0,1]\}^U$, or sometimes simply by $S(U)$.*

$$S_{\mu_A}(U) = \begin{cases} 1, & \mu_A(x) \geq \alpha; \\ 0, & \mu_A(x) \leq \beta; \\ [0,1], & \alpha < \mu_A(x) < \beta. \end{cases} \tag{3}$$

Definition 2.4. *(Soft set theory) [4]: Let U be a universe set. Let E be a set of parameters. Let $F \subseteq U$ be the mapping of E into the set of all subsets of the set U. A pair (F, E) is called soft set*

$$F : E \longrightarrow \mathcal{P}(U), F(x) = \{ (x, f(x)), x \in E, f(x) \in \mathcal{P}(U) \} \tag{4}$$

Definition 2.5. *(Fuzzy soft set theory) [9]: Let U be a universe set. Let E be a set of parameters. Let F_s the all-fuzzy function Subset of U. Let F_u be mapping. $F_u: E \longrightarrow I^U$ such that $F_s = \phi$ if $x \notin E$, F_u is called the fuzzy approximate function of the fuzzy soft set F_s and the value F_u is a set called x-element of the fuzzy soft set $\forall x \in E$. Where F_s is represented by the set of ordered pairs*

$$F_s = \{ (x, F_u(x)), x \in E, Fu \in I^U \} \tag{5}$$

Definition 2.6. *(Possibility Fuzzy soft set theory) [8]: Let $U = \{x_1, x_2, \ldots, x_n\}$ be the universal set of elements and $E = \{e_1, e_2, \ldots, e_n\}$ be the universal set of parameters. Then the pair (U, E) will be called a soft universe. Let $F: E \to I^U$ and μ be a fuzzy subset of E, that is $\mu: E \to I^U$ where I^U is the collection of all fuzzy subsets of U. Let $F_\mu: E \to I^U \times I^U$ be a function defined as follows: $F_\mu(e) = (F(e)(x), \mu(e)(x))$, $\forall x \in U$, then F_μ is called a possibility fuzzy soft set (PFSS) over the soft universe (U, E). For each parameter e_i, $F_\mu(e_i) = (F(e_i)(x), \mu(e_i)(x))$ indicates not only the degree of belongingness of the elements of U in $F_\mu(e_i)$, but also the degree of possibility of belongingness of the elements of U in $F_\mu(e_i)$, which is represented by $\mu(e_i)$. So one can write $F_\mu(e_i)$ as follows:*

$$F_\mu(e_i) = \left\{ \left(\frac{x_1}{F(e_i)(x_1)}, \mu(e_i)(x_1) \right), \left(\frac{x_2}{F(e_i)(x_2)}, \mu(e_i)(x_2) \right), \ldots, \left(\frac{x_n}{F(e_i)(x_n)}, \mu(e_i)(x_n) \right) \right\} \tag{6}$$

Definition 2.7. *(Possibility intuitionistic fuzzy soft set theory) [13]: Let $U = \{x_1, x_2, \ldots, x_n\}$ be the universal set of elements and $E = \{e_1, e_2, \ldots, e_n\}$ be the universal set of parameters. Then the pair (U, E) will be called a soft universe. Let $F: E \to (I \times I)^U \times I^U$ where $(I \times I)^U$ is the collection of all intuitionistic fuzzy subsets of U and I^U is the collection of all* *fuzzy subsets of* U. *Let* p *be a fuzzy subset of* E, *that is,* $p: E \to I^U$ *and let* $F_p: E \to (I \times I)^U \times I^U$ *function defined as follows:* $F_p(e) = (F(e)(x), p(e)(x))$, *where* $F(e)(x) = (\mu(x), v(x))\ \forall x \in U$, *then* F_p *is called a possibility intuitionistic fuzzy soft set (PIFSS in short) over the soft universe* (U, E) . *For each parameter* e_i , $F_p(e_i) = (F(e_i)(x), p(e_i)(x))$, *indicates not only the degree of belongingness of the elements of* U *in* $F(e_i)$, *but also the degree of possibility of belongingness of the elements of* U *in* $F(e_i)$, *which is represented by* $p(e_i)$. *So we can write* $F_p(e_i)$ *as follows:*

$$F_p(e_i) = \left\{\left(\frac{x_1}{F(e_i)(x_1)}, p(e_i)(x_1)\right), \left(\frac{x_2}{F(e_i)(x_2)}, p(e_i)(x_2)\right), \dots, \left(\frac{x_n}{F(e_i)(x_n)}, p(e_i)(x_n)\right)\right\} \tag{7}$$

Definition 2.8. *(Shadow soft set) [14]: Let* U *be a universal set of elements,* (F, E) *be a fuzzy soft set over* U *where* F *is mapping given by* $F: E \to I^U$, E *be the set of parameters and let* $shdw = \{(\alpha_1, \beta_1), (\alpha_2, \beta_2) \dots . (\alpha_m, \beta_m)\}$ *be the shadow parameters set that related to E. Let* $shdw(U)$ *is the set of all shadow subsets on* U. *A pair* $(F, E)_{shdw}$ *is called a shadow soft set over* U, *where* F *is mapping* $F(\alpha_i, \beta_i): E \to shdw(U), \forall i = 1,2, \dots m$. *Thus, we can write* $F(\alpha_i, \beta_i)$ *as follows:*

$$F(\alpha_i, \beta_i)(ei) = \left\{\frac{x_j}{f_i(x_j)}, \forall i = 1,2, \dots m,\ j = 1,2 \dots n\right\}, where\ f_j(x_j) = \begin{cases} 0, & if\ \mu_j(x_j) \le \alpha, \\ 1, & if\ \mu_j(x_j) \ge \beta \\ [0,1], & if\ \alpha \le \mu_j(x_j) \le \beta \end{cases} \tag{2.8}$$

Definition 2.9. (*Possibility shadow soft Set) the following definitions and properties are defined by Alzghoul [15]: Let* U *be the universal set of elements and let* E *be the set of parameters. The pair* (U, E) *are called a soft universe. Let* $F_{Sh}: E \to Sh\,(U)$ *and* μ *be a fuzzy subset of* U, *and* $: E \to I^U$, *where* I^U *is the collection of all fuzzy subsets of* U. *Let* $\left(F^{\mu}_{Sh}(e)\right)^{\beta}_{\alpha}: E \to (I \times I)^U$ *be a function defined as follows: Where* $Sh(U)$ *is all shadow subsets of* U.*Then* $\left(F^{\mu}_{Sh}(e)\right)^{\beta}_{\alpha}$ *is called a Possibility Shadow Soft Set (PSSS in short) over the soft universe* (U, E). *Where* α *and* $\beta \in [0,1]$. *For each parameter* e_i, $\left(F^{\mu}_{Sh}(e_i)\right)^{\beta}_{\alpha} = ((F_{Sh}\,(e_i)(x), \mu\,(e_i)(x))$ *indicates not only the degree of belongingness of the elements of* U *in* $Sh(U)$ *but also the degree of possibility of belongingness of the elements of* U *in* $Sh(U)$ *which is represented by* I^U *so we can write* $\left(F^{\mu}_{Sh}(e)\right)^{\beta}_{\alpha}$ *as follows:*

$$\left(F^{\mu}_{Sh}(e)\right)^{\beta}_{\alpha} = \left\{\left(\frac{x_1}{Sh(U)(x_1)}, \mu(x_1)\right), \left(\frac{x_2}{Sh(U)(x_2)}, \mu(x_2)\right), \dots, \left(\frac{x_n}{Sh(U)(x_n)}, \mu(x_n)\right)\right\} \tag{9}$$

Definition 2.10. *Let* $\left(F^{p}_{Sh}(e)\right)^{\beta}_{\alpha}$ *and* $\left(G^{q}_{Sh}(e)\right)^{\beta}_{\alpha}$ *be two PSSSs over* (U, E). $\left(F^{p}_{Sh}(e)\right)^{\beta}_{\alpha}$ *is said to be a possibility shadow soft subset (PSS subset) of* $\left(G^{q}_{Sh}(e)\right)^{\beta}_{\alpha}$ *and one writes*

$$\left(F^{p}_{Sh}(e)\right)^{\beta}_{\alpha} \subseteq \left(G^{q}_{Sh}(e)\right)^{\beta}_{\alpha}\ if\ \mu(e) \subseteq \mathcal{V}(e)\ for\ all\ e \in E \tag{2.10}$$

Definition 2.11. *A PSSS is said to be a possibility null fuzzy soft set, denoted by* $(\varphi^{\mu}_{Sh})^{\beta}_{\alpha}$ *if* $(\varphi^{\mu}_{Sh})^{\beta}_{\alpha}: E \to Sh(U) \times I^U$ *such that*

$$(\varphi^{\mu}_{Sh})^{\beta}_{\alpha}(e) = (F(e)(x), \mu(e)(x)), \quad \forall e \in E,\ where\ F(e) = 0\ and\ \mu(e) = 0\ , for\ all\ e \in E\ and\ x \in U \tag{11}$$

Definition 2.12. *A PFSSs is said to be a possibility absolute fuzzy soft set, denoted by* A^{P}_{Sh}, *if* $A^{P}_{Sh}: E \to Sh(U) \times I^U$ *such that*

$$A^{P}_{Sh}(e) = (F(e)(x), \mu(e)(x)), \quad \forall e \in E,\ where\ F(e) = 1\ and\ \mu(e) = 1, for\ all\ e \in E\ and\ x \in U \tag{12}$$

Definition 2.13. *Let* $\left(F^{p}_{Sh}(e)\right)^{\beta}_{\alpha}$ *be an PSSS over* (U, E). *Then the complement of* $\left(F^{p}_{Sh}(e)\right)^{\beta}_{\alpha}$ *dented by* $\left(\left(F^{p}_{Sh}(e)\right)^{\beta}_{\alpha}\right)^{c}$ *and defined by*

$$\left(\left(F^{p}_{Sh}(e)\right)^{\beta}_{\alpha}\right)^{c} = (W(e), v(e)), \quad \forall e \in E, \tag{13}$$

such that $W(e) = \tilde{c}(\lambda(e))$ *and* $(e) = c(\psi(e))\ \forall e \in E$, *where* $\tilde{c}$ *is a shadow soft complement and* c *is a fuzzy complement.*

Definition 2.14. *Union of two PSSSs* $\left(F_{Sh}^{p}(e)\right)_{\alpha}^{\beta}$ *and* $\left(G_{Sh}^{q}(e)\right)_{\alpha}^{\beta}$, *denoted by* $\left(F_{Sh}^{p}(e)\right)_{\alpha}^{\beta} \tilde{\cup} \left(G_{Sh}^{q}(e)\right)_{\alpha}^{\beta}$, *is a PFSSs* $H: E \to Sh(U) \times I^{U}$ *defined by*

$$(H_{Sh}^{r})_{\alpha}^{\beta}(e) = \left(H_{Sh}(e)(x), v(e)(x)\right), \quad \forall e \in E, \tag{14}$$

such that $H(e) = s\left(F(e) \tilde{\cup} G(e)\right)$ *and* $v(e) = s\left(\mu(e), \delta(e)\right)$ *where s is an s-norm.*

Definition 2.15. *Intersection of two PSSSs* $\left(F_{Sh}^{p}(e)\right)_{\alpha}^{\beta}$ *and* $\left(G_{Sh}^{q}(e)\right)_{\alpha}^{\beta}$, *denoted by* $\left(F_{Sh}^{p}(e)\right)_{\alpha}^{\beta} \tilde{\cap} \left(G_{Sh}^{q}(e)\right)_{\alpha}^{\beta}$, *is a PFSSs* $H_{v} : E \to \quad Sh(U) \times I^{U}$ *defined by*

$$(H_{Sh}^{r})_{\alpha}^{\beta}(e) = \left(H_{Sh}(e)(x), v(e)(x)\right), \quad \forall e \in E, \tag{15}$$

where $(H_{Sh}^{r})_{\alpha}^{\beta}(e)$ *is a Shadow soft intersection of* $\left(F_{Sh}^{p}(e)\right)_{\alpha}^{\beta}$ *and* $\left(G_{Sh}^{q}(e)\right)_{\alpha}^{\beta}$ *such that* $H(e) = (F(e)) \tilde{\cap} (G(e))$ *and* $v(e) = t(\mu(e)), (\delta(e))$ *where it is a fuzzy t-norm.*

Proposition 2.1. *Let* $\left(F_{Sh}^{p}(e)\right)_{\alpha}^{\beta}$ *be a PSSSs over* (U, E). *Then the following results hold:*

i. $\left(F_{Sh}^{p}(e)\right)_{\alpha}^{\beta} \tilde{\cup} \left(F_{Sh}^{p}(e)\right)_{\alpha}^{\beta} = \left(F_{Sh}^{p}(e)\right)_{\alpha}^{\beta}$,

ii. $\left(F_{Sh}^{p}(e)\right)_{\alpha}^{\beta} \tilde{\cap} \left(F_{Sh}^{p}(e)\right)_{\alpha}^{\beta} = \left(F_{Sh}^{p}(e)\right)_{\alpha}^{\beta}$,

iii. $\left(F_{Sh}^{p}(e)\right)_{\alpha}^{\beta} \tilde{\cup} \left(A_{Sh}^{p}(e)\right)_{\alpha}^{\beta} = \left(A_{Sh}^{p}(e)\right)_{\alpha}^{\beta}$,

iv. $\left(F_{Sh}^{p}(e)\right)_{\alpha}^{\beta} \tilde{\cap} \left(A_{Sh}^{p}(e)\right)_{\alpha}^{\beta} = \left(F_{Sh}^{p}(e)\right)_{\alpha}^{\beta}$,

v. $\left(F_{Sh}^{p}(e)\right)_{\alpha}^{\beta} \tilde{\cup} \left(\varphi_{Sh}^{\mu}\right)_{\alpha}^{\beta} = \left(F_{Sh}^{p}(e)\right)_{\alpha}^{\beta}$,

vi. $\left(F_{Sh}^{p}(e)\right)_{\alpha}^{\beta} \tilde{\cap} \left(\varphi_{Sh}^{\mu}\right)_{\alpha}^{\beta} = \left(\varphi_{Sh}^{\mu}\right)_{\alpha}^{\beta}$.

Proposition 2.17. *Let* $\left(F_{Sh}^{p}(e)\right)_{\alpha}^{\beta}$, $\left(G_{Sh}^{q}(e)\right)_{\alpha}^{\beta}$ *and* $\left(H_{Sh}^{r}(e)\right)_{\alpha}^{\beta}$ *be any three PFSSs over* (U, E), *then the following results hold:*

i. $\left(F_{Sh}^{p}(e)\right)_{\alpha}^{\beta} \tilde{\cup} \left(G_{Sh}^{q}(e)\right)_{\alpha}^{\beta} = \left(G_{Sh}^{q}(e)\right)_{\alpha}^{\beta} \tilde{\cup} \left(F_{Sh}^{p}(e)\right)_{\alpha}^{\beta}$,

ii. $\left(F_{Sh}^{p}(e)\right)_{\alpha}^{\beta} \tilde{\cap} \left(G_{Sh}^{q}(e)\right)_{\alpha}^{\beta} = \left(G_{Sh}^{q}(e)\right)_{\alpha}^{\beta} \tilde{\cap} \left(F_{Sh}^{p}(e)\right)_{\alpha}^{\beta}$,

iii. $\left(F_{Sh}^{p}(e)\right)_{\alpha}^{\beta} \tilde{\cup} \left(\left(G_{Sh}^{q}(e)\right)_{\alpha}^{\beta} \tilde{\cup} \left(H_{Sh}^{r}(e)\right)_{\alpha}^{\beta}\right) = \left(\left(F_{Sh}^{p}(e)\right)_{\alpha}^{\beta} \tilde{\cup} \left(G_{Sh}^{q}(e)\right)_{\alpha}^{\beta}\right) \tilde{\cup} \left(H_{Sh}^{r}(e)\right)_{\alpha}^{\beta}$,

iv. $\left(F_{Sh}^{p}(e)\right)_{\alpha}^{\beta} \tilde{\cap} \left(\left(G_{Sh}^{q}(e)\right)_{\alpha}^{\beta} \tilde{\cap} \left(H_{Sh}^{r}(e)\right)_{\alpha}^{\beta}\right) = \left(\left(F_{Sh}^{p}(e)\right)_{\alpha}^{\beta} \tilde{\cap} \left(G_{Sh}^{q}(e)\right)_{\alpha}^{\beta}\right) \tilde{\cap} \left(H_{Sh}^{r}(e)\right)_{\alpha}^{\beta}$.

Definition 2.18. *If* $\left(\left(F_{Sh}^{p}(e)\right)_{\alpha}^{\beta}, A\right)$ *and* $\left(\left(G_{Sh}^{q}(e)\right)_{\alpha}^{\beta}, B\right)$ *are two PSSSs then "* $\left(\left(F_{Sh}^{p}(e)\right)_{\alpha}^{\beta}, A\right)$ *AND* $\left(\left(G_{Sh}^{q}(e)\right)_{\alpha}^{\beta}, B\right)$ *", denoted by* $\left(\left(F_{Sh}^{p}(e)\right)_{\alpha}^{\beta}, A\right) \wedge \left(\left(G_{Sh}^{q}(e)\right)_{\alpha}^{\beta}, B\right)$ *is defined by*

$$\left(\left(F_{Sh}^{p}(e)\right)_{\alpha}^{\beta}, A\right) \wedge \left(\left(G_{Sh}^{q}(e)\right)_{\alpha}^{\beta}, B\right) = \left(\left(H_{Sh}^{r}(e)\right)_{\alpha}^{\beta}, A \times B\right),$$

where $H_{\lambda}(\alpha, \beta) = \left(H(\alpha, \beta)(x), v(\alpha, \beta)(x)\right)$ *for all* $(\alpha, \beta) \in A \times B$, *such that* $H(\alpha, \beta) = t\left(\left(F_{Sh}^{p}(e)\right)_{\alpha}^{\beta}(\alpha), \left(G_{Sh}^{q}(e)\right)_{\alpha}^{\beta}(\beta)\right)$ *and* $v(\alpha, \beta) = t\left(\mu(\alpha), \delta(\beta)\right)$, *for all* $(\alpha, \beta) \in A \times B$, *where t is t-norm.*

Definition 2.19. *If* $\left(\left(F_{Sh}^{p}(e)\right)_{\alpha}^{\beta}, A\right)$ *and* $\left(\left(G_{Sh}^{q}(e)\right)_{\alpha}^{\beta}, B\right)$ *are two PSSSs then "* $\left(\left(F_{Sh}^{p}(e)\right)_{\alpha}^{\beta}, A\right)$ *OR* $\left(\left(G_{Sh}^{q}(e)\right)_{\alpha}^{\beta}, B\right)$ *", denoted by* $\left(\left(F_{Sh}^{p}(e)\right)_{\alpha}^{\beta}, A\right) \vee \left(\left(G_{Sh}^{q}(e)\right)_{\alpha}^{\beta}, B\right)$ *is defined by*

$$\left(\left(F_{Sh}^{p}(e)\right)_{\alpha}^{\beta}, A\right) \vee \left(\left(G_{Sh}^{q}(e)\right)_{\alpha}^{\beta}, B\right) = \left(\left(H_{Sh}^{r}(e)\right)_{\alpha}^{\beta}, A \times B\right),$$

where $\left(H_{Sh}^{r}(e)\right)_{\alpha}^{\beta}(\alpha, \beta) = \left(H(\alpha, \beta)(x), v(\alpha, \beta)(x)\right)$ *for all* $(\alpha, \beta) \in A \times B$, *such that* $H(\alpha, \beta) = s\left(\left(F_{Sh}^{p}(e)\right)_{\alpha}^{\beta}(\alpha), \left(G_{Sh}^{q}(e)\right)_{\alpha}^{\beta}(\beta)\right)$ *and* $v(\alpha, \beta) = s\left(\mu(\alpha), \delta(\beta)\right)$, *for all* $(\alpha, \beta) \in A \times B$, *where s is s-norm.*

3 Intuitionistic Possibility Shadow Soft Sets

In this section we generalize the concept of possibility shadow soft sets as introduced by Alzghoul [15]. In our generalization of possibility shadow soft sets, a possibility of each element in the universe is attached with the parameterization of shadow sets while defining an Intuitionistic possibility shadow soft sets.

Definition 3.1. *Let $U = \{x_1, x_2, \dots, x_n\}$ be the universal set of elements and $E = \{e_1, e_2, \dots, e_n\}$ be the universal set of parameters. Then the pair (U, E) will be called a soft universe. (F, E) is called a shadow soft set over U. Let $\left(F_{Sh}^{p}\right)_{\alpha}^{\beta}: E \to Sh(U) \times (I \times I)^{U}$ where $(I \times I)^{U}$ is the collection of all intuitionistic fuzzy subsets of U and $Sh(U)$ is the collection of all shadow subsets of U. Let p be an intuitionistic fuzzy subset of E, that is, $p: E \to (I \times I)^{U}$ and let $\left(F_{Sh}^{p}(e)\right)_{\alpha}^{\beta}: E \to Sh(U) \times (I \times I)^{U}$ be a function defined as:*

$$\left(F_{Sh}^{p}(e)\right)_{\alpha}^{\beta} = \big(\gamma(e)(x), p(e)(x)\big), \text{ where } p(e)(x) = \big(\mu(x), \nu(x)\big), \ \forall x \in U,$$

then $\left(F_{Sh}^{p}(e)\right)_{\alpha}^{\beta}$ is called an ***Intuitionistic possibility shadow soft set*** *(IPSSS) over the soft universe (U, E) where α and $\beta \in [0,1]$. For each parameter e_i, $\left(F_{Sh}^{p}(e_i)\right)_{\alpha}^{\beta} = \big(\gamma(e_i)(x), p(e_i)(x)\big)$ indicates not only the degree of belongingness and non-belongingness of the elements of U in $Sh(U)$, but also the degree of possibility of belongingness of the elements of U in $Sh(U)$, and also the degree of possibility of non-belongingness of the elements of U in $Sh(U)$, which is represented by $p(e_i)$. So we can write $\left(F_{Sh}^{p}\right)_{\alpha}^{\beta}(e)$ as follows:*

$$\left(F_{Sh}^{p}(e)\right)_{\alpha}^{\beta} = \left\{\left(\frac{x_1}{Sh(U)(x_1)}, p(x_1)\right), \left(\frac{x_2}{Sh(U)(x_2)}, p(x_2)\right), \dots, \left(\frac{x_n}{Sh(U)(x_n)}, p(x_n)\right)\right\}.$$

Definition 3.2. *Let $\left(F_{Sh}^{p}\right)_{\alpha}^{\beta}(e)$ and $\left(G_{Sh}^{q}\right)_{\alpha}^{\beta}$ be two IPSSSs over (U, E). Then $\left(F_{Sh}^{p}\right)_{\alpha}^{\beta}(e)$ is said to be an intuitionistic possibility shadow soft subset (IPSS subset) of $\left(G_{Sh}^{q}\right)_{\alpha}^{\beta}(e)$ and we write $\left(F_{Sh}^{p}\right)_{\alpha}^{\beta}(e) \subseteq \left(G_{Sh}^{q}\right)_{\alpha}^{\beta}(e)$ if:*

i. *$p(e)$ is an Intuitionistic fuzzy subset of $q(e)$, for all $e \in E$,*
ii. *$\delta(e)$ is a shadow soft subset of $\rho(e)$, for all $e \in E$.*

Definition 3.3. *Let $\left(F_{Sh}^{p}\right)_{\alpha}^{\beta}$ and $\left(G_{Sh}^{q}\right)_{\alpha}^{\beta}$ be two IPSSSs over (U, E). Then $\left(F_{Sh}^{p}\right)_{\alpha}^{\beta}$ and $\left(G_{Sh}^{q}\right)_{\alpha}^{\beta}$ are said to be equal and we write $\left(F_{Sh}^{p}\right)_{\alpha}^{\beta} = \left(G_{Sh}^{q}\right)_{\alpha}^{\beta}$ if F_{Sh}^{p} is an IPSS subset of $\left(G_{Sh}^{q}\right)_{\alpha}^{\beta}$ and $\left(G_{Sh}^{q}\right)_{\alpha}^{\beta}$ is a IPSS subset of $\left(F_{Sh}^{p}\right)_{\alpha}^{\beta}$ the following conditions are satisfied:*

i. *$p(e)$ is equal to $q(e)$, for all $e \in E$,*
ii. *$\left(F_{Sh}^{p}\right)_{\alpha}^{\beta}(e)$ is equal to $\left(G_{Sh}^{q}\right)_{\alpha}^{\beta}(e)$, for all $e \in E$.*

Definition 3.4. *An IPSSS is said to be a null intuitionistic possibility shadow soft set with respect to α and β, denoted by $\left(\varphi_{Sh}^{p}\right)_{\alpha}^{\beta}$, such that:*

$$\left(\varphi_{Sh}^{p}\right)_{\alpha}^{\beta}(e) = \big(\gamma(e)(x), p(e)(x)\big), \quad \forall e \in E,$$

where $\gamma(e) = 0$ and $p(e) = \big(0, \nu(e)\big)$, for all $e \in E$.

Definition 3.5. *An IPSSS is said to be an absolute intuitionistic possibility shadow soft set with respect to α and β, denoted by A_{Sh}^{P}, such that:*

$$A_{Sh}^{P}(e) = \big(\gamma(e)(x), p(e)(x)\big), \quad \forall e \in E,$$

where $F(e) = 1$ and $P(e) = (0,1)$, for all $e \in E$.

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