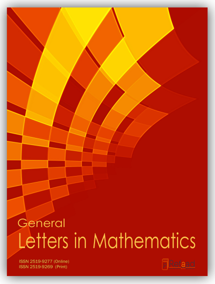

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Some Properties of P_p-Compact Spaces

Shadya M. Mershkhan[*1], **Baravan A. Asaad** [2]

[1,2] Department of Mathematics, Faculty of Science, University of Zakho, IRAQ
[2] Department of Computer Science, College of Science, Cihan University-Duhok, IRAQ

[1]mathshadya@gmail.com,[2]baravan.asaad@uoz.edu.krd

Abstract. In this paper, the concepts of P_p-compact spaces by using nets, filter base and P_p-complete accumulation points are introduced and studied.

Keywords: *P_p-open sets, P_p-continuous functions, P_p-compact spaces, strongly compact spaces, P_p-complete accumulation points.*
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1 Introduction

Compactness and properties closely related to compactness play an important role in the applications of general topology to real analysis and functional analysis. Mashhour et al. [11] defined preopen sets and precontinuous functions. In 2014, Khalaf and Mershkhan [9] introduced P_p-open sets, which are stronger than preopen sets, in order to investigate the characterization of P_p-continuous functions. Jafari [4] defined the concept of θ-compact spaces. Mashhour et al. [12] introduced the concept of strongly compact spaces. The aim of this paper is giving some characterizations of P_p-compact spaces in terms of nets and filter bases. The class of P_p-compact spaces lies strictly between the classes of strongly compact space and θ-compact space, but it is not comparable with compact space. We also introduce the notion of P_p-complete accumulation points by which we give some characterizations of P_p-compact spaces.

2 Preliminaries

Throughout this paper, (X, τ) and (Y, σ) (or simply X and Y) stand for topological spaces with no separation axioms are assumed unless otherwise stated. For a subset A of X, the closure of A and the interior of A will be denoted by $Cl(A)$ and $Int(A)$, respectively. $A \subseteq X$ is said to be preopen [11] (resp., semi-open [10] and α-open [17]) if $A \subset Int(Cl(A))$ (resp., $A \subseteq Cl(Int(A))$ and $A \subset Int(Cl(Int(A)))$). The complement of a preopen (resp., semi-open) set is preclosed (resp., semi-closed). $A \subseteq X$ is called preclopen [6] if A is both preopen and peclosed. Also $A \subseteq X$ is called θ-open [21] if for each $x \in A$, there exists an open set G such that $x \in G \subseteq Cl(G) \subseteq A$. A preopen subset A of X is called P_p-open [9] (resp., P_S-open [7]) if for each $x \in A$, there exists a preclosed (resp., semi-closed) set F such that $x \in F \subseteq A$. The complement of a P_p-open set is a P_p-closed. A subset A of X is pre-regular open [15] if A =pInt(pCl(A)). The family of all preopen (resp., pre-regular open, θ-open, P_p-open and P_S-open) of X is denoted

*Corresponding author. Mershkhan and Asaad [1] mathshadya@gmail.com

by $PO(X)$ (resp., $PRO(X)$, $\theta O(X)$, $P_pO(X)$ and $P_SO(X)$). The preclosure (resp., P_p-closure) of A, denoted by $pCl(A)$ (resp., $P_pCl(A)$) is defined as the intersection of all preclosed (resp., P_p-closed) sets. The preinterior (resp., P_p-interior) of A, denoted by $pInt(A)$ (resp., $P_pInt(A)$) is defined as the union of all preopen (resp., P_p-open) sets.

Definition 1. *A space X is said to be:*

1. *locally indiscrete [3] if every open subset of X is closed.*
2. *pre-T_1 [6] if for each pair of distinct points x, y of X, there exist two preopen sets one containing x but not y and the other containing y but not x.*
3. *pre-regular [19] if for each preclosed F and each point $x \notin F$, there exist disjoint preopen sets U and V such that $x \in U$ and $F \subseteq V$.*

Lemma 2. *[19] A space X is pre-regular if and only if for each $x \in X$ and each $G \in PO(X)$ there exists $H \in PO(X)$ such that $x \in H \subseteq pCl(H) \subseteq G$.*

Proposition 3. *The following statements are true:*

1. *If X is pre-T_1, then $PO(X) = P_pO(X)$ [9].*
2. *If X is pre-regular, then $\tau \subseteq P_pO(X)$ [9].*
3. *If X is locally indiscrete, then $\tau = P_SO(X)$ [7].*
4. *If X is locally indiscrete, then $\tau \subseteq P_pO(X)$ [9].*
5. *If X is locally indiscrete, then $PO(X) = P_pO(X)$ [9].*

Lemma 4. *[9] Let Y be a subspace of a space X and $A \subseteq X$. Then the following properties are hold:*

1. *If $A \in P_pO(Y)$ and either Y is either preclopen or $Y \in PRO(X)$, then $A \in P_pO(X)$.*
2. *If $A \in P_pO(X)$ and Y is both α-open and preclosed subset of X, then $A \cap Y \in P_pO(Y)$.*

Definition 5. *A filter base $\Im$ is said to be p-converges [5] (resp., θ-converges [2], P_S-converges [8]) to a point $x \in X$ if for every preopen (resp., θ-open and P_S-open) set V containing x, there exists an $F \in \Im$ such that $F \subseteq V$.*

Definition 6. *[21] A filter base $\Im$ is said to be δ-converges to a point $x \in X$ if for every open set V containing x, there exists an $F \in \Im$ such that $F \subseteq Int(Cl(V))$.*

Definition 7. *A filter base $\Im$ is said to be p-accumulates [5] (resp., θ-accumulates [2] and P_S-accumulates [8]) to a point $x \in X$ if $F \cap V \neq \phi$ for every preopen (resp., θ-open and P_S-open) set V containing x and every $F \in \Im$.*

Definition 8. *A space X is said to be strongly compact [12] (resp., θ-compact [4]) if every preopen (resp., θ-open) cover of X has a finite subcover.*

Definition 9. *[1] A space X is said to be p-closed if for every preopen cover $\{V_\alpha : \alpha \in \Delta\}$ of X, there exists a finite subset Δ_0 of Δ such that $X = \cup\{pCl(V_\alpha) : \alpha \in \Delta_0\}$.*

Definition 10. *A subset A of a space X is said to be N-closed [18] relative to X if for every cover $\{V_\alpha : \alpha \in \Delta\}$ of A by open sets of X, there exists a finite subset Δ_0 of Δ such that $A \subseteq \cup\{Int(Cl(V_\alpha)) : \alpha \in \Delta_0\}$. A space X is said to be nearly compact [20] if X is N-closed relative to X.*

Definition 11. *A function $f : X \to Y$ is called precontinuous [11] (resp., P_p-continuous [9]) at a point $x \in X$ if for each open set V of Y containing $f(x)$, there exists a preopen (resp. P_p-open) set U of X containing x such that $f(U) \subseteq V$.*

Definition 12. *A function $f : X \to Y$ is called almost precontinuous [16] (resp., almost P_p-continuous [14]) at a point $x \in X$ if for each open set V of Y containing $f(x)$, there exists a preopen (resp., P_p-open) set U of X containing x such that $f(U) \subseteq IntCl(V)$.*

Theorem 13. *[13] If $f : X \to Y$ is a continuous and open function and V is a P_p-open set of Y, then $f^{-1}(V)$ is a P_p-open set of X.*

3 P_p-Compact Spaces

In this section, we introduce a new class of spaces called P_p-compact spaces and study some of its properties.

Definition 14. *A filter base $\Im$ on X P_p-converges to a point $x \in X$ if for every P_p-open set V containing x, there exists an $F \in \Im$ such that $F \subseteq V$.*

Definition 15. *A filter base $\Im$ on X P_p-accumulates to a point $x \in X$ if $F \cap V \neq \phi$, for every P_p-open set V containing x and every $F \in \Im$.*

Remark 16. *A filter base $\Im$ P_p-accumulates at x if and only if $x \in \bigcap\{P_pCl(F) : F \in \Im\}$. Clearly, if a filter base $\Im$ P_p-converges to a point $x \in X$, then $\Im$ P_p-accumulates to a point x.*

The converse of Remark 16 is not true in general as shown by the following example.

Example 17. *Consider $X = \{a, b, c, d\}$ with $\tau = \{\phi, \{a\}, \{a, b\}, \{c, d\}, \{a, c, d\}, X\}$ and $\Im = \{\{b, c\}, \{a, b, c\}, \{b, c, d\}, X\}$. Then $P_pO(X) = \{\phi, \{c\}, \{d\}, \{c, d\}, \{a, b\},$ $\{a, b, c\}, \{a, b, d\}, X\}$. Thus $\Im$ P_p-accumulates to a, but $\Im$ does not P_p-converges to a, because the set $\{a, b\}$ is a P_p-open set containing a, and there is no an $F \in \Im$ such that $F \subseteq \{a, b\}$.*

Theorem 18. *Let $\Im$ be a filter base on X. Then there exists a filter base finer than $\{u_x\}$, where $\{u_x\}$ is the family of P_p-open sets of X containing x if and only if there exists a filter base $\Im_1$ finer than $\Im$ and P_p-converges to x.*

Proof. Let $\Im_1$ be a filter base which is finer than both $\Im$ and $\{u_x\}$. Then $\Im$ P_p-converges to x since it contains $\{u_x\}$. Conversely, let $\Im_1$ be the filter base which is finer than $\Im$ and converges to x. Then $\Im$ must contain $\{u_x\}$ by definition. □

Corollary 19. *If $\Im$ is a maximal filter base on X, then $\Im$ P_p-converges to a point $x \in X$ if and only if $\Im$ P_p-ccumulates to a point x.*

Proof. Let $\Im$ be a maximal filter base in X and P_p-accumulates to a point $x \in X$,then by Theorem 18, there exists a filter base $\Im_1$ finer than $\Im$ and P_p-converges to x. But $\Im$ is a maximal filter base. Thus it is P_p-convergent to x. □

Proposition 20. *Let $\Im$ be a filter base on X. If $\Im$ p-converges to a point $x \in X$, then $\Im$ P_p-converges to a point x.*

Proof. Suppose that $\Im$ p-converges to a point $x \in X$. Let V be any P_p-open set containing x, then V is preopen set containing x. Since $\Im$ p-converges to a point $x \in X$, there exists an $F \in \Im$ such that $F \subseteq V$. This shows that $\Im$ P_p-converges to a point x. □

Proposition 21. *Let $\Im$ be a filter base on X. If $\Im$ p-accumulates to a point $x \in X$, then $\Im$ P_p-accumulates to a point x.*

Proof. The proof is similar to the Proposition 20. □

The following examples show that the converses of Proposition 20 and Proposition 21 is not true in general.

Example 22. *Consider $X = \{a, b, c, d\}$ with $\tau = \{\phi, \{b\}, \{a, d\}, \{a, b, d\}, X\}$ and $\Im = \{\{b, c\}, \{a, b, c\}, \{b, c, d\}, X\}$. Then $PO(X) = \{\phi, \{a\}, \{b\}, \{d\}, \{a, b\},$ $\{a, d\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \{b, c, d\}, X\}$ and $P_pO(X) = \{\phi, \{a\}, \{d\}, \{a, d\},$ $\{a, b, c\}, \{b, c, d\}, X\}$. Thus $\Im$ P_p-converges to b, but $\Im$ does not p-converges to b, because the set $\{b\}$ is preopen containing b, and there is no an $F \in \Im$ such that $F \subseteq \{b\}$.*

Example 23. *In Example 17, $PO(X) = \{\phi, \{a\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, X\}$. Thus $\Im$ P_p-accumulates to a, but $\Im$ does not P-accumulates to a, because the set $\{a\}$ is preopen containing a, and there exists an $F \in \Im$ such that $F \cap \{a\} = \phi$.*

Proposition 24. *Let $\Im$ be a filter base on X. If $\Im$ P_p-converges (resp., P_p-accumulates) to a point $x \in X$, then $\Im$ θ-converges (resp., θ-accumulates) to a point x.*

Proof. Obvious from the fact that every θ-open set is P_p-open. □

Proposition 25. *Let $\Im$ be a filter base in a locally indiscrete space X. If $\Im$ P_p-converges (resp., P_p-accumulates) to a point $x \in X$, then $\Im$ P_S-converges (resp., P_S-accumulates) to a point x.*

Proof. Suppose that $\Im$ be a filter base P_p-converges (resp., P_p-accumulates) to a point $x \in X$. Let V be any P_S-open set containing x. Since X is a locally indiscrete space, then by Proposition 3 (3) and (5), V is a P_p-open set containing x. Since $\Im$ P_p-converges (resp., P_p-accumulates) to a point $x \in X$, then there exists an $F \in \Im$ such that $F \subseteq V$ (resp., $F \cap V \neq \phi$). This shows that $\Im$ P_S-converges (resp., P_S-accumulates) to a point x. □

The converses of Proposition 25 and Proposition 24 are not true as shown by the next example.

Example 26. *Consider $X = \{a, b, c, d\}$ with $\tau = \{\phi, \{a\}, \{b, c, d\}, X\}$ and $\Im = \{\{b, c\}, \{a, b, c\}, \{b, c, d\}, X\}$. Then $P_pO(X)$ is a discrete topology on X and $\theta O(X) = P_SO(X) = \tau$. Thus $\Im$ P_S-converges (resp., θ-converges) to b, but $\Im$ does not P_p-converges to b, because the set $\{b\}$ is P_p-open containing b, and there is no an $F \in \Im$ such that $F \subseteq \{b\}$. Also, $\Im$ P_S-accumulates (resp., θ-accumulates) to d, but $\Im$ does not P_p-accumulates to d, because the set $\{d\}$ is P_p-open containing d, and there exists an $F \in \Im$ such that $F \cap \{d\} = \phi$.*

Proposition 27. *Let $\Im$ be a filter base on X and E be any P_p-closed set containing $x \in X$. If there exists an $F \in \Im$ such that $F \subseteq E$, then $\Im$ P_p-converges to a point x.*

Proof. Let V be any P_p-open set containing $x \in X$, then for each $x \in V$, there exists a P_p-closed set E such that $x \in E \subseteq V$. By hypothesis, there exists an $F \in \Im$ such that $F \subseteq E \subseteq V$ which implies that $F \subseteq V$. Hence $\Im$ P_p-converges to a point x. □

Proposition 28. *Let $\Im$ be a filter base on X and E be any P_p-closed set containing x. If there exists an $F \in \Im$ such that $F \cap E \neq \phi$, then $\Im$ P_p-accumulates to a point x.*

Proof. The proof is similar to the Proposition 27. □

Theorem 29. *If a function $f : X \to Y$ is P_p-continuous (resp., almost P_p-continuous), then for each point $x \in X$ and each filter base $\Im$ in X P_p-converging to x, the filter base $f(\Im)$ is convergent (resp., δ-convergent) to $f(x)$.*

Proof. Suppose that $x \in X$ and $\Im$ is any filter base in X which P_p-converges to x. By the P_p-continuity (resp., almost P_p-continuity) of f, for any open set V in Y containing $f(x)$, there exists $U \in P_pO(X)$ containing x such that $f(U) \subseteq V$(resp., $f(U) \subseteq Int(Cl(V))$). But $\Im$ is P_p-convergent to x in X, then there exists an $F \in \Im$ such that $F \subseteq U$. It follows that $f(F) \subseteq V$(resp., $f(F) \subseteq Int(Cl(V))$). This means that $f(\Im)$ is convergent (resp., δ-convergent) to $f(x)$. □

Definition 30. *A space X is said to be P_p-compact if for every P_p-open cover $\{V_\alpha : \alpha \in \Delta\}$ of X, there exists a finite subset Δ_0 of Δ such that $X = \cup\{V_\alpha : \alpha \in \Delta_0\}$.*

Proposition 31. *If every preclosed cover of a space X has a finite subcover, then X is P_p-compact.*

Proof. Let $\{V_\alpha : \alpha \in \Delta\}$ be any P_p-open cover of X, then for each $x \in X$, there exists $\alpha \in \Delta_0$, $x \in V_{\alpha(x)}$, there exists a preclosed set $F_{\alpha(x)}$ such that $x \in F_{\alpha(x)} \subseteq V_{\alpha(x)}$. therefore, the family $\{F_{\alpha(x)} : x \in X\}$ is a preclosed cover of X, then by hypothesis, this family has a finite subcover such that $X = \{F_{\alpha(x_i)} : i = 1, 2, , n\} \subseteq \{ V_{\alpha(x_i)} : i = 1, 2, , n\}$. Therefore, $X = \{V_{\alpha(x_i)} : i = 1, 2, , n\}$. Hence, X is P_p-compact. □

Lemma 32. *If X is strongly compact, then it is P_p-compact.*

Proof. The proof is straightforward because every P_p-open set is preopen. □

The converse of lemma 32 is not true as shown by the next example.

Example 33. *Let $\mathbb{R}$ be the set of real numbers with topology $\tau = \{\phi, G \subseteq \mathbb{R}$ such that $1 \in G\}$. Hence, X is P_p-compact, but it is not strongly compact.*

Lemma 34. *Every P_p-compact space is θ-compact.*

Proof. Obvious from the fact that every θ-open set is P_p-open. □

The converse of Lemma 34 is not true as shown by the next example.

Example 35. *Let $\mathbb{R}$ be the set of real numbers with topology $\tau = \{\phi, \mathbb{R}, \{1\}, \mathbb{R} \setminus \{1\}\}$. Then X is θ-compact, but it is not P_p-compact.*

From lemma 32 and lemma 34, the following diagram is obtained:

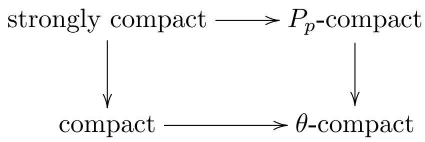

Diagram 3.1

In general, P_p-compact spaces and compact spaces are not comparable as shown by the following two examples.

Example 36. *In Example 33, X is P_p-compact, but it is not compact.*

Example 37. *In Example 35, X is compact, but it is not P_p-compact.*

Proposition 38. *Let X be a locally indiscrete space. If X is P_p-compact, then it is P_S-compact.*

Proof. Follows from Proposition 3 (3) and (4). □

The converse of Proposition 38 is not true as shown by the next example.

Example 39. *In Example 35, X is P_S-compact, but it is not P_p-compact.*

Lemma 40. *Let X be either a pre-regular or a locally indiscrete space. If X is P_p-compact, then it is compact.*

Proof. Follows from Proposition 3 (2) and (4). □

Lemma 41. *Every locally indiscrete and strongly compact space is P_p-compact.*

Proof. Follows from Proposition 3 (5). □

Theorem 42. *Every pre-T_1 and P_p-compact space is strongly compact.*

Proof. Suppose that X is a pre-T_1 and a P_p-compact space. Let $\{V_\alpha : \alpha \in \Delta\}$ be any preopen cover of X. Then for every $x \in X$, there exists $\alpha(x) \in \Delta$ such that $x \in V_{\alpha(x)}$. Since X is pre-T_1, by Proposition 3 (1), the family $\{V_\alpha : \alpha \in \Delta\}$ is a P_p-open cover of X. Since X is P_p-compact, there exists a finite subset Δ_0 of Δ in X such that $X = \cup\{V_\alpha : \alpha \in \Delta_0\}$. Hence, X is strongly compact. □

Proposition 43. *If X is a pre-regular and p-closed space, then it is P_p-compact.*

Proof. Let $\{V_\alpha : \alpha \in \Delta\}$ be any P_p-open cover of X, then V_α is preopen for each $\alpha \in \Delta$. Since X is a pre-regular, by lemma 2, for each $x \in X$ and $V_{\alpha(x)}$, there exists a preopen set G_x such that $x \in G_x \subseteq pCl(G_x) \subseteq V_{\alpha(x)}$. Then the family $\{G_x : x \in X\}$ is a preopen cover of X. Since X is a p-closed space, then there exists a subfamily $\{G_{x_i} : i = 1, 2, ..., n\}$ such that $X = \cup_{i=1}^{n} pCl(G_{x_i}) \subseteq \cup_{i=1}^{n} V_{\alpha(x_i)}$. Thus X is P_p-compact. □

Theorem 44. *For any space X. The following statements are equivalent:*

1. *X is P_p-compact,*
2. *For any P_p-open cover $\{V_\alpha : \alpha \in \Delta\}$ of X, there exists a finite subset Δ_0 of Δ such that $X = \cup\{V_\alpha : \alpha \in \Delta_0\}$,*
3. *Every maximal filter base $\Im$ in X P_p-converges to some point $x \in X$,*
4. *Every filter base $\Im$ in X P_p-accumulates to some point $x \in X$,*

5. *For every family $\{F_\alpha : \alpha \in \Delta\}$ of P_p-closed subsets of X such that $\cap\{F_\alpha : \alpha \in \Delta\} = \phi$, there exists a finite subset Δ_0 of Δ such that $\cap\{F_\alpha : \alpha \in \Delta_0\} = \phi$.*

Proof. (1) $\Rightarrow$ (2) Straightforward.
(2) $\Rightarrow$ (3) Suppose that for every P_p-open cover $\{V_\alpha : \alpha \in \Delta\}$ of X, there exists a finite subset Δ_0 of Δ such that $X = \cup\{V_\alpha : \alpha \in \Delta_0\}$ and let $\Im = \{F_\alpha : \alpha \in \Delta\}$ be a maximal filter base. Suppose that $\Im$ does not P_p-converges to any point of X. Since $\Im$ is maximal, by Corollary 19, $\Im$ does not P_p-accumulates to any point of X. This implies that for every $x \in X$ there exist P_p-open set V_x and $F_{\alpha(x)} \in \Im$ such that $F_{\alpha(x)} \cap V_x = \phi$. The family $\{V_x : x \in X\}$ is a P_p-open cover of X and by hypothesis, there exists a finite number of points $x_1, x_2, ..., x_n$ of X such that $X = \cup\{V_{(x_i)} : i = 1, 2, ..., n\}$. Since $\Im$ is a filter base on X, there exists a $F_0 \in \Im$ such that $F_0 \subseteq \cap\{F_{\alpha(x_i)} : i = 1, 2, ..., n\}$. Hence, $F_0 \cap V_{\alpha(x_i)} = \phi$ for $i = 1, 2, ..., n$ which implies that $F_0 \cap (\cup\{V_{(x_i)} : i = 1, 2, ..., n\}) = F_0 \cap X = \phi$. Therefore, we obtain $F_0 = \phi$. Which contradicts the fact that $\Im \neq \phi$. Thus $\Im$ is P_p-converges to some point $x \in X$.
(3) $\Rightarrow$ (4) Let $\Im$ be any filter base on X. Then, there exists a maximal filter base $\Im_0$ such that $\Im \subseteq \Im_0$. By hypothesis, $\Im_0$ P_p-converges to some point $x \in X$. For every $F \in \Im$ and P_p-open set V containing x, there exists $F_0 \in \Im_0$ such that $F_0 \subseteq V$. Hence $\phi \neq F_0 \cap F \subseteq V \cap F$. This shows that $\Im$ P_p-accumulates at x.
(4) $\Rightarrow$ (5) Let $\{F_\alpha : \alpha \in \Delta\}$ be a family of P_p-closed subsets of X such that $\cap\{F_\alpha : \alpha \in \Delta\} = \phi$. If possible, suppose that every finite subfamily $\cap\{F_{\alpha_i} : i = 1, 2, ..., n\} \neq \phi$. Therefore, $\Im = A \subseteq Y \subseteq X$ form a filter base on X. By hypothesis, $\Im$ P_p-accumulates to some point $x \in X$. This implies that for every P_p-pen set V containing x, $F_\alpha \cap V \neq \phi$, for every $F_\alpha \in \Im$ and every $\alpha \in \Delta$. Since $x \notin \cap F_\alpha$, there exist an $\alpha_0 \in \Delta$ such that $x \notin F_{\alpha_0}$. Hence, $X \setminus F_{\alpha_0}$ is a P_p-open set containing x and $F_{\alpha_0} \cap X \setminus F_{\alpha_0} = \phi$. Which contradicts the fact that $\Im$ P_p-accumulates to x. Therefore, the assertion in (5) is true.
(5) $\Rightarrow$ (1) Let $\{V_\alpha : \alpha \in \Delta\}$ be a P_p-open cover of X. Then $\{X \setminus V_\alpha : \alpha \in \Delta\}$ is a family of P_p-closed subsets of X such that $\cap\{X \setminus V_\alpha : \alpha \in \Delta\} = \phi$. By hypothesis, there exists a finite subset Δ_0 of Δ such that $\cap\{X \setminus V_\alpha : \alpha \in \Delta_0\} = \phi$. Hence, $X = \cup\{V_\alpha : \alpha \in \Delta_0\}$. This shows that X is P_p-compact. □

4 P_p-Sets and P_p-Compact Subspaces

In this section, new classes of space called P_p-set and P_p-compact subspace are introduced.

Definition 45. *A subset A of a space X is said to be P_p-set (resp., P_p-compact subspace) if for every cover $\{V_\alpha : \alpha \in \Delta\}$ of A by P_p-open subsets of X (resp., by P_p-open subsets of A), there exists a finite subset Δ_0 of Δ such that $A \subseteq \cup\{V_\alpha : \alpha \in \Delta_0\}$ (resp., $A = \cup\{V_\alpha : \alpha \in \Delta_0\}$).*

Lemma 46. *A subset A of a space X is a P_p-set (resp., a P_p-compact subspace) if and only if for every cover of A by P_p-open sets of X (resp., by P_p-open sets of A) has a finite subcover.*

Proof. The proof follows directly from Definition 45. □

Now several equivalent conditions to P_p-sets (resp., P_p-compact subspaces) of spaces are given as well as giving some other conditions such that each of which makes a given space a P_p-compact space.

Theorem 47. *Let A be a subset of a space X. If every cover of A by preclosed subsets of X (resp., by preclosed subsets of A) has a finite subcover, then A is a P_p-set (resp., a P_p-compact subspace).*

Proof. Let $\{V_\alpha : \alpha \in \Delta\}$ be a cover of A by P_p-open subset of X (resp., by P_p-open subsets of A). Then for each $x \in X$, there exists $\alpha \in \Delta_0$, $x \in V_{\alpha(x)}$, there exists a preclosed set $F_{\alpha(x)}$ such that $x \in F_{\alpha(x)} \subseteq V_{\alpha(x)}$. Hence, the family $\{F_{\alpha(x)} : x \in X\}$ is a cover of A by preclosed subsets of X (resp., by preclosed subsets of A). Then by hypothesis, this family has a finite subcover such that $A \subseteq \cup\{F_{\alpha(x_i)} : i = 1, 2, , n\} \subseteq \cup\{V_{\alpha(x_i)} : i = 1, 2, , n\}$ (resp., $A = \{F_{\alpha(x_i)} : i = 1, 2, , n\} \subseteq \cup\{V_{\alpha(x_i)} : i = 1, 2, , n\}$. Therefore, $A \subseteq \cup\{V_{\alpha(x_i)} : i = 1, 2, , n\}$ (resp., $A = \cup\{V_{\alpha(x_i)} : i = 1, 2, , n\}$. Hence, A is P_p-set (resp., P_p-compact subspace). □

Theorem 48. *For any space X. The following statements are equivalent:*

1. *A is P_p-set (resp., P_p-compact subspace),*
2. *Every maximal filter base $\Im$ on X which meets A P_p-converges to some point of A,*
3. *Every filter base $\Im$ on X which meets A P_p-accumulates to some point $x \in X$,*

4. *For every family $\{F_\alpha : \alpha \in \Delta\}$ of P_p-closed subsets of X such that $[\cap\{F_\alpha : \alpha \in \Delta\}] \cap A = \phi$, there exists a finite subset Δ_0 of Δ such that $[\cap\{F_\alpha : \alpha \in \Delta_0\}] \cap A = \phi$.*

Proof. Similar to Theorem 44. □

Theorem 49. *A space X is P_p-compact if and only if every proper P_p-closed set of X is P_p-set.*

Proof. **Necessity:** Let F be any proper P_p-closed set of X. Let $\{V_\alpha : \alpha \in \Delta\}$ be a cover of F and $V_\alpha \in P_pO(X)$ for every $\alpha \in \Delta$. Since F is a P_p-closed set, then $X \setminus F$ is a P_p-open set. Thus the family $\{V_\alpha : \alpha \in \Delta\} \cup (X \setminus F)$ is a P_p-open cover of X. Since X is P_p-compact, there exists a finite subset Δ_0 of Δ such that $X = \cup\{V_\alpha : \alpha \in \Delta_0\} \cup (X \setminus F)$. Therefore, we obtain $F \subseteq \cup\{V_\alpha : \alpha \in \Delta_0\}$. Hence, F is P_p-set.
Sufficiency: Let $\{V_\alpha : \alpha \in \Delta\}$ be a cover of X and $V_\alpha \in P_pO(X)$ for every $\alpha \in \Delta$. Suppose that $X \neq V_{\alpha_0} \neq \phi$ for every $\alpha_0 \in \Delta$. Then $X \setminus V_{\alpha_0}$ is a proper P_p-closed subset of X. Therefore, by hypothesis, there exists a finite subset Δ_0 of Δ such that $X \setminus V_{\alpha_0} \subseteq \cup\{V_\alpha : \alpha \in \Delta_0\}$. Therefore, we obtain $X = \cup\{V_\alpha : \alpha \in \Delta_0 \cup \{\alpha_0\}\}$. Which shows that X is P_p-compact. □

Theorem 50. *If a space X is P_p-compact and A is both preclopen and P_p-closed subset of X, then A is a P_p-compact subspace.*

Proof. Let $\{A_\alpha : \alpha \in \Delta\}$ be any cover of A by P_p-open set of A. Since A is preclopen, by Lemma 4 (1), $A_\alpha \in P_pO(X)$ for each $\alpha \in \Delta$. Since A is a P_p-closed subset of X, then $(X \setminus A) \in P_pO(X)$ and $\{A_\alpha : \alpha \in \Delta\} \cup (X \setminus A) = X$ and $\{A_\alpha : \alpha \in \Delta\} \cup (X \setminus A)$ forms a P_p-open cover of X. Since X is P_p-compact, there exists a finite subset Δ_0 of Δ such that $X = \cup\{A_\alpha : \alpha \in \Delta_0\} \cup (X \setminus A)$. Hence, $A = \cup\{A_\alpha : \alpha \in \Delta_0\}$. Therefore, A is a P_p-compact subspace. □

Lemma 51. *If a space X is P_p-compact and A is both pre-regular open and P_p-closed subset of X, then A is A P_p-compact subspace.*

Proof. Follows from Theorem 50 and Lemma 4 (1). □

Theorem 52. *If there exists either a proper α-open and a proper preclosed subset A of a space X such that A and $X \setminus A$ are P_p-compact subspace, then X is also P_p-compact.*

Proof. Let $\{V_\lambda : \lambda \in \Lambda\}$ be any P_p-open cover of X. Since A is an α-open and a preclosed subset of X, then for every $\lambda \in \Lambda$, by Lemma 4 (2), we have $A \cap V_\lambda \in P_pO(A)$. Therefore, $\{A \cap V_\lambda : \lambda \in \Lambda\}$ is a P_p-open cover of A. Since A is a P_p-compact subspace, there exists a finite subset Λ_0 of Λ such that $A = \cup\{A \cap V_\lambda : \lambda \in \Lambda_0\}$. Therefore, we have $A \subseteq \cup\{V_\lambda : \lambda \in \Lambda_0\}$. Since A is an α-open and a preclosed subset of X, then $X \setminus A$ is also an α-open and a preclosed. By the same way, we can find a finite subset Λ_1 of Λ such that $X \setminus A \subseteq \cup\{V_\lambda : \lambda \in \Lambda_1\}$. Hence $X = \cup\{V_\lambda : \lambda \in \Lambda_0 \cup \Lambda_1\}$. This shows that X is P_p-compact. □

Theorem 53. *Let A be any subset of a space X such that A and $X \setminus A$ are P_p-set of X. Then X is also P_p-set.*

Proof. Let $\{V_\alpha : \alpha \in \Delta\}$ be any P_p-open cover of $X = A \cup X \setminus A$. Then $\{V_\alpha : \alpha \in \Delta\}$ is an P_p-open cover of A and $X \setminus A$. Therefore, there exist finite subsets Δ_0 and Δ_1 of Δ such that $A \subseteq \cup\{V_\alpha : \alpha \in \Delta_0\}$ and $X \setminus A \subseteq \cup\{V_\alpha : \alpha \in \Delta_1\}$. Thus $X = A \cup X \setminus A \subseteq \cup\{V_\alpha : \alpha \in \Delta_0 \cup \Delta_1\}$. This completes the proof. □

Theorem 54. *If a preclopen set G of a space X is a P_p-set, then G is a P_p-compact subspace.*

Proof. Suppose that G is a preclopen and a P_p-set. Let $\{V_\alpha : \alpha \in \Delta\}$ be a cover of G and $V_\alpha \in P_pO(G)$ for every $\alpha \in \Delta$. Since G is a preclopen set, then by Lemma 4 (1), we have $V_\alpha \in P_pO(X)$ for every $\alpha \in \Delta$. Since G is P_p-set, there exists a finite subset Δ_0 of Δ such that $G \subseteq \cup\{V_\alpha : \alpha \in \Delta_0\}$, which implies that G is P_p-compact subspace. □

Corollary 55. *If a pre-regular open set G of a space X is a P_p-set, then G is a P_p-compact subspace.*

Proof. This is an immediate consequence of Theorem 54 and Lemma 4 (1). □

Theorem 56. *If G is an α-open, a P_p-closed of a space X and G is P_p-compact subspace, then G is P_p-set.*

Proof. Suppose that G is an α-open and a P_p-closed, and $\{V_\lambda : \lambda \in \Lambda\}$ be a cover of G and $V_\alpha \in P_pO(X)$ for every $\lambda \in \Lambda$. Since G is an α-open and a P_p-closed, then for every $\lambda \in \Lambda$, by Lemma 4 (2), we have $G \cap V_\lambda : \lambda \in P_pO(G)$. Therefore, the family $\{G \cap V_\lambda : \lambda \in \Lambda\}$ is a P_p-open cover of G. Since G is a P_p-compact subspace, there exists a finite subset Λ_0 of Λ such that $G = \cup\{G \cap V_\lambda : \lambda \in \Lambda_0\}$. Therefore, $G \subseteq \cup\{V_\lambda : \lambda \in \Lambda_0\}$, which implies that G is P_p-set. □

Theorem 57. *Let A and B be subsets of a space X. If A is P_p-closed and B is P_p-set, then $A \cap B$ is P_p-set.*

Proof. Let $\{V_\alpha : \alpha \in \Delta\}$ be any cover of $A \cap B$ by P_p-open subsets of X. Since A is a P_p-closed set, then $X \setminus A$ is P_p-open. Thus $B \subseteq \cup\{V_\alpha : \alpha \in \Delta\} \cup (X \setminus A)$ and the family $\{V_\alpha : \alpha \in \Delta\} \cup (X \setminus A)$ is a P_p-open cover of B. Since B is a P_p-set, then there exists a finite subset Δ_0 of Δ such that $B \subseteq \cup\{V_\alpha : \alpha \in \Delta_0\} \cup (X \setminus A)$. Therefore, we obtain that $A \cap B \subseteq \cup\{V_\alpha : \alpha \in \Delta_0\}$. Hence, $A \cap B$ is a P_p-set. □

Corollary 58. *The finite union of a P_p-set (resp., a P_p-compact subspace) of X is a P_p-set (resp., a P_p-compact subspace).*

Proof. Straightforward. □

Theorem 59. *Let B be P_p-set of X and G be θ-open subset of a space X such that $G \subseteq B$. Then, $B \setminus G$ is P_p-set.*

Proof. Obvious. □

5 Results on Images of P_p-Compactness

Theorem 60. *If a function $f : X \to Y$ is P_p-continuous (resp., almost P_p-continuous) and A is P_p-set, then $f(A)$ is compact (resp., N-closed) relative to Y.*

Proof. Let $\{G_\alpha : \alpha \in \Delta\}$ be any cover of $f(A)$ by open sets of Y. For each $x \in A$, there exists an $\alpha(x) \in \Delta$ such that $f(x) \in G_{\alpha(x)}$. Since f is P_p-continuous (resp., almost P_p-continuous), there exists a P_p-open set U_x of X containing x such that $f(U_x) \subseteq G_{\alpha(x)}$ $(resp., f(U_x) \subseteq Int(Cl(G_{\alpha(x)})))$. Then the family $\{U_\alpha : x \in A\}$ is a P_p-open cover of A. For some finite subset A_0 of A, we have $A \subseteq \cup\{U_x : x \in A_0\}$. Therefore, $f(A) \subseteq \cup \{G_{\alpha(x)} : x \in A_0\}$ (resp., $f(A) \subseteq \cup\{Int(Cl(G_{\alpha(x)})) : x \in A_0\}$). This shows that $f(A)$ is compact (resp., N-closed) relative to Y. □

Corollary 61. *If $f : X \to Y$ is a P_p-continuous (resp., almost P_p-continuous) surjection function and X is a P_p-compact, then Y is compact (resp., nearly compact)*

Proposition 62. *If $f : X \to Y$ is a P_p-continuous (resp., almost P_p-continuous), A is a P_p-set and F is a P_p-closed subset of X, then $f(A \cap F)$ is compact (resp., N-closed) relative to Y.*

Proof. Follows from Theorem 60 and Theorem 57. □

Proposition 63. *If $f : X \to Y$ is a precontinuous (resp., almost precontinuous) surjection function and X is a pre-T_1 and P_p-compact space, then Y is compact (resp., nearly compact).*

Proof. Follows from Theorem 60 and Theorem 42. □

Proposition 64. *If $f : X \to Y$ is a precontinuous (resp., almost precontinuous) surjection function and X is a locally indiscrete and P_p-compact space, then Y is compact (resp., nearly compact).*

Proof. Follows from Theorem 60 and Lemma 41. □

Proposition 65. *If $f : X \to Y$ is a continuous (resp., almost P_p-continuous) surjection function and X is a locally indiscrete and P_p-compact space, then Y is compact (resp., nearly compact).*

Proof. Follows from Theorem 60 and Lemma 40. □

Proposition 66. *If $f : X \to Y$ is a continuous (resp., almost P_p-continuous) surjection function and X is a pre-regular and P_p-compact space, then Y is compact (resp., nearly compact).*

Proof. Follows from Theorem 60 and Lemma 40. □

Theorem 67. *If $f : X \to Y$ is a continuous and open function. If A is a P_p-set, then $f(A)$ is a P_p-set.*

Proof. Let $\{V_\alpha : \alpha \in \Delta\}$ be any cover of $f(A)$ by P_p-open sets of Y. Since f is continuous and open function. By Theorem 13, $\{f^{-1}(V_\alpha) : \alpha \in \Delta\}$ is a cover of A by P_p-open sets of X. Since A is P_p-set, there exists a finite subset Δ_0 of Δ such that $A \subseteq \cup\{f^{-1}(V_\alpha) : \alpha \in \Delta_0\}$. Thus, we have $f(A) \subseteq \cup\{V_\alpha : \alpha \in \Delta_0\}$. This shows that $f(A)$ is P_p-set. □

Corollary 68. *If X is a P_p-compact space and $f : X \to Y$ is a continuous and open surjection function, then Y is P_p-compact.*

6 Characterization of P_p-compact spaces

Definition 69. *A point x in X is said to be P_p-complete accumulation point of a subset A of X if $Card(A \cap U) = Card(A)$ for each $U \in P_pO(X)$, where $Card(A)$ denotes the cardinality of A.*

Definition 70. *In a space X, a point x is said to be a P_p-adherent point of a filter base $\Im$ on X if it lies in the P_p-closure of all sets of $\Im$.*

Theorem 71. *A space X is P_p-compact if and only if each infinite subset of X has a P_p-complete accumulation point.*

Proof. Let the space X be P_p-compact and S be an infinite subset of X. Let K be the set of points x in X which are not P_p-complete accumulation points of S. Now it is obvious that for each point x in K, we are able to find $U_{(x)} \in P_pO(X, x)$ such that $Card(S \cap U_{(x)}) \neq Card(S)$. If K is the whole space, then $E = \{U_{(x)} : x \in X\}$ is a P_p-open cover of X. By hypothesis, X is P_p-compact. Therefore, there exists a finite subcover $\Psi = \{U_{(x_i)} : i = 1, 2, ..., n\}$ such that $S \subseteq \cup\{U_{(x_i)} \cap S : i = 1, 2, ..., n\}$. Then, $Card(S) = \max\ \{Card(U_{(x_i)} \cap S) : i = 1, 2, ..., n\}$, which does not agree with what we assumed. This implies that S has a P_p-complete accumulation point. Now assume that X is not a P_p-compact and that every infinite subset S of X has a P_p-complete accumulation point in X. It follows that there exists a cover Θ with no finite subcover. Set δ=min$\{Card(\Xi) : \Xi \subseteq \Theta$, where Ξ is a P_p-open cover of $X\}$. Fix $\Psi \subseteq \Theta$, for which $Card(\Psi) = \delta$ and $\cup\{U : U \in \ \Psi\} = X$. Let $\mathbb{N}$ denotes the set of natural numbers, then by hypothesis $\delta \geq Card(\mathbb{N})$ by well-ordering of Ψ. By some minimal well-ordering " $\sim$ " , suppose that U is any member of Ψ. By minimal well-ordering " $\sim$ ", we have $Card(\{V : V \in \Psi, V \sim U\}) < Card(\{V : V \in \Psi\})$. Since Ψ can not have any subcover with cardinality less that δ, then for each $U \in \Psi$, we have $X \neq \cup\{V : V \in \Psi, V \sim U\}$. For each $U \in \Psi$, choose a point $x(U) \in X \setminus \cup\{V \cup \{x(V)\} : V \in \Psi, V \sim U\}$. We are always able to do this, if not, one can choose a cover of smaller cardinality from Ψ. If $H = \{x(U) : U \in \Psi\}$, then to finish the proof, we will show that H has no P_p-complete accumulation point in X. Suppose that z is a point of the space X. Since Ψ is a P_p-open cover of X, then z is a point of some set W in Ψ. By the fact that $U \sim W$, we have $x(U) \in W$. It follows that $T = \{U : U \in \Psi$ and $x(U) \in W\} \subseteq \{V : V \in V \sim W\}$. But $Card(T) < \delta$. Therefore, $Card(H \cap W) < \delta$. But $Card(H) = \delta \geq Card(N)$, since for two distinct points U and W in Ψ, we have $x(U) \neq x(W)$. This means that H has no P_p-complete accumulation point in X which contradicts our assumptions. Therefore, X is a P_p-compact. □

Theorem 72. *For a space X, the following are equivalent:*

1. *X is P_p-compact.*
2. *Every net in X with well-ordered directed set as its domain accumulates to some point of X.*

Proof. (1) $\Rightarrow$ (2) Suppose that X is a P_p-compact and $\xi = \{x_\alpha : \alpha \in \Delta\}$ a net with a well-ordered set Δ as domain. Assume that ξ has no P_p-adherent point in X. Then for each point x in X, there exist $V_{(x)} \in P_pO(X, x)$ and an $\alpha(x) \in \Delta$ such that $V_{(x)} \cap \{x_\alpha : \alpha \geq \alpha(x)\} = \phi$. This implies that $\{x_\alpha : \alpha \geq \alpha(x)\}$ is a subset of $X \setminus V_{(x)}$. Then the collection $\omega = \{V_{(x)} : x \in X\}$ is a P_p-open cover of X. By hypothesis of theorem, X is P_p-compact and so ω has a finite subfamily $\{V_{(x_i)} : i = 1, 2, ..., n\}$ such that $X = \cup\{V_{(x_i)} : i = 1, 2, ..., n\}$. Suppose that the corresponding elements of Δ be $\{\alpha(x_i)\}$ where $i = 1, 2, , n$, since Δ is well-ordered and $\{\alpha(x_i)\}$ where $i = 1, 2, , n$ is finite. The largest elements of $\{\alpha(x_i)\}$ exists. Suppose it is $\{\alpha(x_i)\}$. Then for $\gamma \geq \{\alpha_{(x_i)}\}$. We have $\{x_\delta : \delta \geq \gamma\} \subseteq \bigcap_{i=1}^{n}(X \setminus V_{(x_i)}) = X \setminus \bigcup_{i=1}^{n} V_{(x_i)} = \phi$. Which is impossible. This shows that ξ has at least one P_p-adherent point in X.

(2) $\Rightarrow$ (1) Now, it is enough to prove that each infinite subset has a P_p-complete accumulation point by utilizing above theorem. Suppose that $S \subseteq X$ is an infinite subset of X. According to Zorns Lemma, the infinite set S can be well-ordered. This means that we can assume S to be a net with a domain which is a well ordered index set. It follows that S has P_p- adherent point z. Therefore, z is a P_p-complete accumulation point of S. This shows that X is a P_p-compact. □

Theorem 73. *A space X is a P_p-compact if and only if each family of a P_p-closed subsets of X with the finite intersection property has a non-empty intersection.*

Proof. Given a collection ω of subsets of X. Let $\nu = \{X \setminus \varpi : \varpi \in \omega\}$ be the collection of their complements. Then the following statements hold.

1. ω is the collection of P_p-open sets if and only if ν is a collection of P_p-closed sets.

2. the collection ω covers of X if and only if the intersection $\bigcap_{\upsilon\in\nu}(\nu)$ of all the elements of ν is non empty.

3. The finite sub collection $\{\omega_n, .., \omega_n\}$ of ω covers X if and only if the intersection of the corresponding elements $\upsilon_i = X \setminus \omega_i$ of ν is empty.
The statement (1) is trivial. While the statement (2) and (3) follows from De-Morgan Law $X \setminus \bigcup_{\alpha\in j}(\nu_\alpha) = \bigcap_{\alpha\in j}(X \setminus \nu_\alpha)$. The proof of theorem now proceeds in two steps. Taking the contra positive of the theorem and the complement. The statement X is a P_p-compact is equivalent to: Given any collection of ω P_p-open subsets of X, if ω covers X, then some finite sub collection of ω covers X. This statement is equivalent to its contra positive, Which is the following.
Given any collection ω of P_p-open sets, if no finite sub collection ω of covers X, then ω does not cover X. Letting ν be as earlier, the collection $\{X \setminus W : W \in \omega\}$, and applying (1) to (3), we see that this statement is in turn equivalent to the following.
Given any collection ν of P_p-closed sets, if every finite intersection of elements of ν is non empty. This is just the condition of our theorem.

□

Theorem 74. *A space X is a P_p-compact if and only if each filter base in X has at least one a P_p-adherent point.*

Proof. Suppose that X is P_p-compact and $\Im = \{F_\alpha : \alpha \in \Delta\}$ is a filter base in it. Since all finite intersections of F_α's are nonempty. It follows that all finite intersections of $P_pCl(F_\alpha)$'s are also nonempty. Now, it follows from Theorem 73 that $\bigcap_{\alpha\in\Delta} P_pCl(F_\alpha)$ is nonempty. This means that $\Im$ has at least one P_p- adherent point. Now, suppose that $\Im$ is any family of P_p-closed sets. Let each finite intersection be nonempty. The set F_α with their finite intersection establish the filter base $\Im$. Therefore, $\Im$ P_p-accumulates to some point z in X. It follows that $z \in \bigcap_{\alpha\in\Delta} F_\alpha$. Now, we have by Theorem 72, that X is a P_p-compact. □

Theorem 75. *A space X is a P_p-compact if and only if each filter base on X, with at most one P_p-adherent point, is a P_p-convergent.*

Proof. Suppose that X is a P_p-compact, x is a point of X, and $\Im$ is a filter base on X. The P_p-adherent of $\Im$ is a subset of $\{X\}$. Then the P_p-adherent of $\Im$ is equal to $\{X\}$, by Theorem 74. Assume that there exists a $V \in P_pO(X, x)$ such that for all $F \in \Im$, $F \cap (X \setminus V)$ is nonempty. Then $\Psi = \{F \setminus V : F \in \Im\}$ is a filter base on X. It follows that the P_p-adherence of Ψ is nonempty. However, $\bigcap_{F\in\Im} P_pCl(F \setminus V) \subseteq (\bigcap_{F\in\Im} P_pClF) \cap (X \setminus V) = \{X\} \cap (X \setminus V) = \phi$. But this is a contradiction. Hence, for each $V \in P_pO(X, x)$, there exist $F \in \Im$ with $F \subseteq V$. This shows that $\Im$ P_p-converges to x. To prove the converse, it suffices to show that each filter base in X has at least one P_p-accumulation point. Assume that $\Im$ is a filter base on X with no P_p-adherent point. By hypothesis $\Im$ P_p-converges to some point z in X. Suppose that F_α is an arbitrary element of $\Im$. Then for each $V \in P_pO(X, z)$, there exists an $F_\beta \in \Im$ such that $F_\beta \subseteq V$. Since $\Im$ is a filter base, there exists a γ such that $F_\gamma \subseteq F_\alpha \cap F_\beta \subseteq F_\alpha \cap V$, where F_γ is a nonempty. This means that $F_\alpha \cap V$ is nonempty for every $V \in P_pO(X, z)$ and correspondingly for each α, z is a point of $P_pCl(F_\alpha)$. It follows that $z \in \bigcap_\alpha P_pCl(F_\alpha)$. Therefore, z is P_p-dherent point of $\Im$. Which is contradiction. This shows that X is P_p-compact. □

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