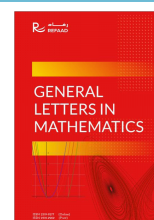




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Forecasting of Covid-19 deaths in South Africa using the autoregressive integrated moving average time series model

Musyoka Kinyili^{a,*}, Maurice Wanyonyi^b

^aDepartment of Mathematics and Applied Mathematics, University of the Western Cape, Private Bag X17 Bellville 7535, South Africa.

^bDepartment of mathematics and Statistics, University of Embu, P. O Box 6–60100 Embu, Kenya.

Abstract

Covid-19 epidemic continues to escalate globally posing life threats to humans. Time series modeling plays a key role for the prediction of data-driven scenarios. A case for Covid-19 pandemic future numbers occurrence is one of the open forecasting scenario for application of the time series modeling. We applied the Autoregressive Integrated Moving Average (ARIMA) model to forecast the possible numbers of Covid-19 deaths in the Republic of South Africa using the previously reported data for a period of 17 months (May 2020 to September 2021). We adapted the Box-Jenkins' methodology to step-by-step achieve the entire forecasting process. We identified the MA(1) (ARIMA(0,0,1)) as the best model based on the Akaike Information Criterion and the Bayesian Information Criterion. The forecasting done at 95% confidence interval for a period of 7 months (October 1, 2021 to April 31, 2022) indicated that the Covid-19 associated deaths in South Africa would slightly increase during the month of October 2021 but remain constant throughout the entire prediction period. ©2021 All rights reserved.

Keywords: ARIMA model application, Covid-19 pandemic, Covid-19-associated deaths, Time series forecasting.

1. Introduction

The Covid-19 pandemic whose originator pathogen is the Severe Acute Respiratory Syndrome Coronavirus-2 (SARS-CoV-2) [1, 2, 3, 4, 6] has been devastating globally since its first report at Wuhan, Hubei province of China in the late December 2019 [4, 7]. The disease spreads quickly and easily majorly via respiratory droplets produced during coughing, sneezing, talking and/or breathing of a Covid-19 infected individual(s) [7, 8, 9, 10, 11, 12, 13, 14]. This feature of the disease made it to spread in most regions of the world within a very short period of time. Reports in [1, 5] indicate that the SARS-CoV-2 is among the many different types of Corona-viruses which are known to circulate among animals and can sometimes spillover to humans. Among these zoonotic Corona-viruses whose outbreak has been experienced before the Covid-19 are the Severe Acute Respiratory syndrome (SARS) and the Middle East Respiratory Syndrome (MERS) [1, 5, 14, 15]. However, these diseases did not take that long to clear from the human host and did not spread in the entire world unlike the Covid-19 epidemic which has broken the record.

In Africa, the first Covid-19 positive case was reported on February 14, 2020 in Cairo city the capital of Egypt by the Ministry of Health and Population [5, 14, 16]. In South Africa, the National Institute for Communicable Diseases (NICD) confirmed the first Covid-19 positive case on March 5, 2020 [7, 8, 11, 17].

*Corresponding author

Email address: davismusyooo@gmail.com (Musyoka Kinyili)

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Since April 12, 2020, South Africa has been leading with the top numbers of Covid-19 active cases and hence becoming the leading Covid-19 core in Africa [5]. For instance, [1] reports that as of 20th day of July 2020, South Africa had recorded a total of 364,324 Covid-19 positive cases, with 5,033 fatalities. This represented about 51% of the Covid-19 cases and 33% of the deaths on the whole of Africa continent. As of October 5, 2020, South Africa had 681,289 Covid-19 confirmed cases and 16,976 deaths. As of January 6, 2021, the total confirmed Covid-19 cases were 1,149,591 with 31,368 fatalities and as of March 11, 2021 the country confirmed 1,525,648 cumulative cases with 51,110 deaths [14]. Keen observations reveal that the rate of deaths due to Covid-19 in South Africa has been significantly high. On June 14, 2020, South Africa was ranked the second position globally in terms of the percentage daily deaths [1]. During the writing of this paper (September 30, 2021) South Africa had recorded 87,626 Covid-19 deaths out of 2,902,672 cumulative infections [18].

Although a lot of modeling works on Covid-19 has so far been accomplished, most of these works have focused a lot on impact analysis of various intervention strategies to reduce the Covid-19 infections while overlooking on the deaths resulting from the Covid-19 pandemic. Time series modeling is among the best statistical tools for the prediction of data-driven scenarios. For instance, [19] used the Autoregressive Integrated Moving Average (ARIMA) time series model to predict the Covid-19 cases using the dataset for the previously reported positive cases for the epidemic. In [20], the first order moving average time series model was selected as the best prediction model and applied to forecast the Covid-19 death cases in Chile. In [21], one month forecasting for the Covid-19 casualties cases using hybrid ARIMA model was carried out. More works applying the ARIMA time series modeling were also done in [22, 23]. Furthermore, comparative studies on the prediction accuracy of the ARIMA and the Holt-Winters time series models were done in [24, 25, 26, 27, 28]. The ARIMA model was found to have better prediction accuracy unlike the Holt-Winters model.

This work focuses on the prediction of Covid-19 associated deaths in South Africa using the Autoregressive Integrated Moving Average (ARIMA) time series model. This forecasting will be significant for more serious precautions to be taken if the Covid-19 continues spreading for unprecedented period of time especially in South Africa. The country has so far experienced three waves of Covid-19 epidemic and there are precautions on the possibility of the outbreak of the fourth wave probably in December 2021. Since our prediction covers a period of 7 months (October 1, 2021 to April 31, 2022), this would serve better for Covid-19 deaths situation advisory for necessary measures to be adopted.

We organize the rest of the paper as follows: Section 2 presents the materials and methods. Section 3 gives the results and discussion and Section 4 concludes the paper.

2. Materials and Methods

2.1. Data Collection and Analysis

Freely available public data of Covid-19 reported deaths of the Republic of South Africa that was obtained from the Department of Health online resource and news portal on Covid-19 [18]. The acquired data covered a period of 17 months running from May 2020 to September 2021. We adapted the ARIMA time series analysis technique to analyze the monthly Covid-19 deaths data. The calibration of the ARIMA model was carried out using the Maximum Likelihood Estimation algorithm implemented in R statistical software. We further applied the information criterion for the selection of the model and used the following R functions and packages for the analysis: *ndiffs*, *auto.arima*, *coefstest*, *acf*, *pacf*, *adf.test*, *Box.test*, *forecast*, *tseries*, *lmtest* and *ggplot2* [29].

2.2. Autoregressive Integrated Moving Average (ARIMA) Model

The ARIMA model, also called the Box-Jenkins model is theoretically the utmost comprehensive model that forecasts a time series and was designed by Box and Jenkins [30]. The predictor variables are totally disregarded in the ARIMA model but it considers its past values and the error terms in forecasting. The ARIMA consists of the Autoregressive (AR), the Integrated (I), and the Moving Average (MA). The

ARIMA model is mostly applicable where data shows sign of lack of stationarity and it may be made stationary through alterations like introducing logarithms and differencing. The ARIMA accounts for historical data that can be split into an autoregressive process which depends on its past [31]; the integrated (I) process, which is responsible for making the series to be stationary and thus enabling forecasting; and finally, the MA process of the errors in the prediction, that is the lengthier the past information is, the better the forecasts are as it progresses. The process has to be stationary in order to fit the ARIMA model.

2.2.1. Components of ARIMA Model

We briefly describe the main components of the ARIMA model. The first component is the Autoregressive (AR) which is a process that normally expresses the response variable as a function of its history [32]. The p^{th} -order autoregressive process is expressed in the form

$$y_t = \alpha_1 y_{t-1} + \alpha_2 y_{t-2} + \dots + \alpha_p y_{t-p} + e_t \quad (2.1)$$

Where, X_t denotes the stationary present value that is forecasted at time t , y_{t-1}, \dots, y_{t-p} denotes the response variables at time $t-1, \dots, t-p$, while e_t is the pure random process. Using a backshift operator B^p , the AR(p) model can be represented by

$$e_t = (1 - \alpha_1 B - \alpha_2 B^2 - \alpha_3 B^3 - \dots - \alpha_p B^p) y_t. \quad (2.2)$$

The second component is the Moving Average (MA) that is a process in which MA (q) denotes entries up to order q that uses past errors to forecast present variable [32] thus we have

$$y_t = \beta_0 e_t + \beta_1 e_{t-1} + \beta_2 e_{t-2} + \dots + \beta_q e_{t-q} \quad (2.3)$$

where q denotes the number of lags in the MA process, $\beta_1, \beta_2, \dots, \beta_q$ represents the parameters to be estimated and e_t is the white noise with a mean of zero and a variance of σ^2 .

The MA operator is given by

$$\phi(B) = (\beta_0 + B^1 \beta_1 + B^2 \beta_2 + \dots + B^q \beta_q). \quad (2.4)$$

Merging the aforementioned first and the second components up to order q , gives the Autoregressive Moving Average process denoted by ARMA (p, q). Thus representation of the ARMA (p, q) model is

$$y_t = \beta_1 e_{t-1} + \dots + \beta_q e_{t-q} + \alpha_1 y_{t-1} + \dots + \alpha_p y_{t-p} + e_t \quad (2.5)$$

where, $\beta_1 e_{t-1} + \dots + \beta_q e_{t-q}$ is the MA (q), and $\alpha_1 y_{t-1} + \dots + \alpha_p y_{t-p}$ is the AR (p). In order to alleviate the non-stationary challenge for achievement of the stationary in the ARMA model, the notion of integration with differencing factor is introduced [32]. This gives rise to the ARIMA model.

2.3. Box-Jenkins Methodology

The Box-Jenkins methodology for time series modelling entails: Model identification, parameter estimation and model selection, model validity, and forecasting [30]. We briefly give a discussion on each of the iterative procedure performed for the forecasting process.

2.3.1. Model Identification

We applied the augmented dickey fuller (ADF) test on the time series in order to determine the presence of stationarity before making a choice on the specific model. We further designed an Autocorrelation Function (ACF) and the Partial Autocorrelation Function (PACF) for identification of the order of the Moving Average (MA) and the Autoregressive (AR) processes.

2.3.2. Parameter Estimation and Model Selection

We adapted the Maximum Likelihood Estimation technique to estimate the parameters of the model by maximizing the likelihood of the observed data. The Likelihood function in standard Gaussian form is

$$\text{LogL} = \frac{T}{2} \log(2\pi) - \frac{T}{2} \log \sigma^2 - \frac{1}{2\sigma^2} \sum_{t=1}^T \varepsilon_t^2 \quad (2.6)$$

Where, T is the time for $t = 1, \dots, T$, while ε_t and σ denote error and standard deviation respectively. The model with maximum log Likelihood was chosen based on the Akaike Information Criterion (AIC).

2.3.3. Model Validity

This step involves determining whether the model fits to data. The normality of the model was tested by applying the Q-Q plot and the Histogram for the residuals [33]. We applied the Ljung-Box test for the verification of the existence of serial autocorrelation between the lags. The significant difference implies that the data is uncorrelated. Otherwise, there is dependence in the series. The Ljung Box test is computed by

$$Q(m) = n(n+2) \sum_{j=1}^m \tilde{\rho}_j^2 / (n-j)^3 \quad (2.7)$$

Where, n is the sample size, $\tilde{\rho}_j$ is the sample autocorrelation for lag j and m are the total number of lags. Here, the null hypothesis is rejected if $Q(m) > (1 - \alpha)$ where, $Q(m)$ is the quantile of χ_m^2 . The hypothesis tested are, H_0 : data is distributed independently versus, H_1 : at least two sets of data are correlated.

2.3.4. Forecasting

Forecasting is the final step of the Box-Jenkins methodology. It involves obtaining future information using past information available. For any model to be applied for prediction, it must pass the validity test otherwise it cannot be used

3. Results and Discussion

3.1. Time series plot

Figure 1 shows the time series plot for Covid-19 Deaths in South Africa for a period of 17 months running from May 2020 to September 2021. From the plot, we observe that the series seems to be stationary due to absence of trend. Now that the series was stationary, differencing of the data was not necessary.

Since the ARIMA time series model requires the data to be stationary, we further applied the Augmented Dickey Fuller (ADF) to identify if the data was stationary before using the data to fit the model. Table 1 shows the results of the ADF test. The p-value of the test is less than the significance value ($\alpha = 0.05$) and this suggests that we fail to reject the null hypothesis which states that the series is non stationary and conclude that there is significant evidence that the series is stationary. Therefore, differencing process is not necessary for this case.

Table 1: Augmented Dickey Fuller test.

Test Statistics	Lag Order	p-value
-3.1755	2	0.0125

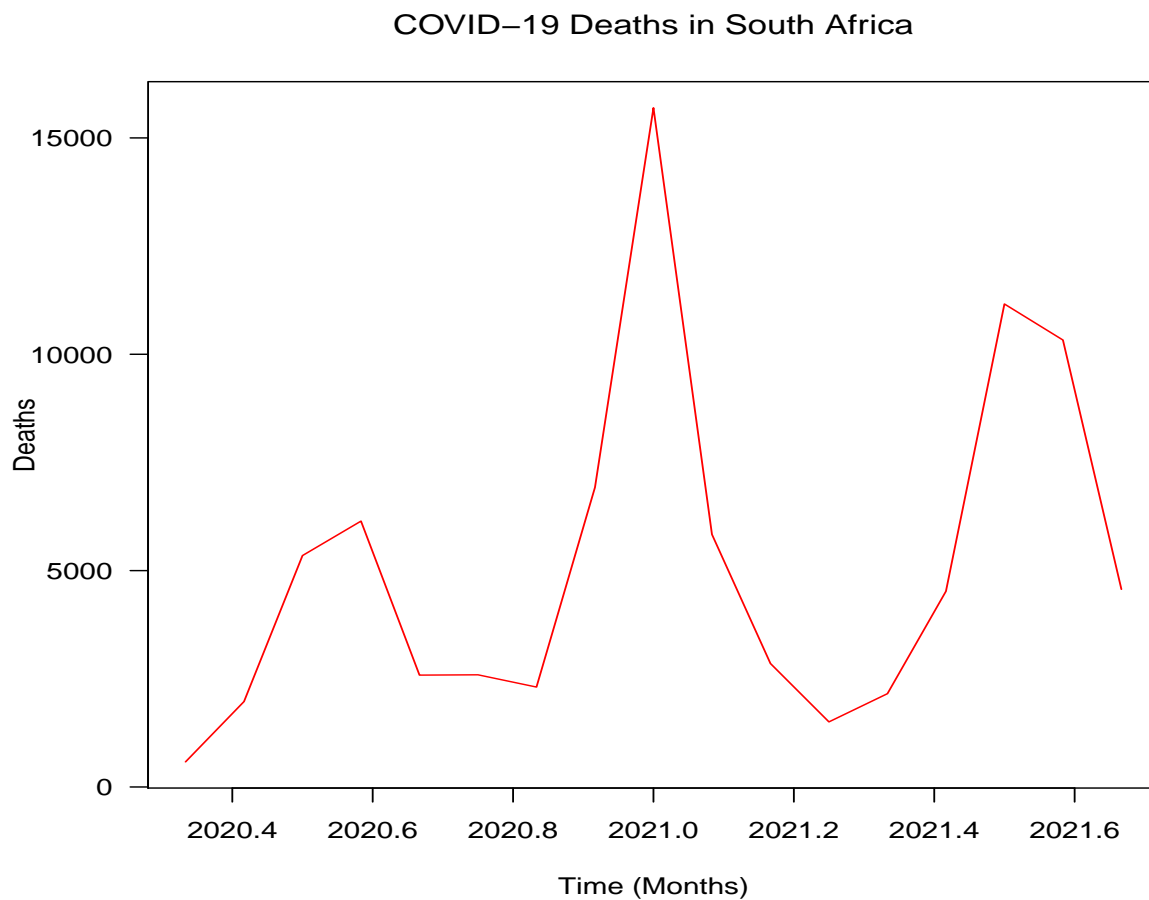


Figure 1: Time series plot for the Covid-19 deaths in South Africa (May 2020 to September 2021).

3.2. Model Identification

The Autocorrelation Function (ACF) and the Partial Autocorrelation Function are applied to identify the order of Moving Average (MA) process and Autoregressive (AR) process respectively. This procedure was performed and the output is depicted by Figure 2. The figure indicates that the ACF cuts off at lag 1 implying that the MA order is 1. The PACF plot shows that the AR order is 0 and since the data was not differenced to achieve stationarity, then the ARIMA (p, d, q) is given by ARIMA (0, 0, 1) which is the first order moving average process, i.e., MA (1) model.

3.3. Model Estimation and Selection

The parameters of the model were estimated using the Maximum Likelihood Estimation algorithm. We fitted the model to the time series data and obtained the parameters estimates as $\sigma^2 = 12992554$ and log-likelihood = -162.48. With the help of the Akaike Information Criterion (AIC) and the Bayesian Information criterion of 330.96 and 333.46 respectively, we were able to select the best ARIMA model. Table 2 shows the regression coefficients of the selected model with MA (1) as the only coefficient at 5% significance level.

The regression equation of the first order moving average process was obtained as,

$$z_t = 4977.198\epsilon_t + 0.565\epsilon_{t-1} \quad (3.1)$$

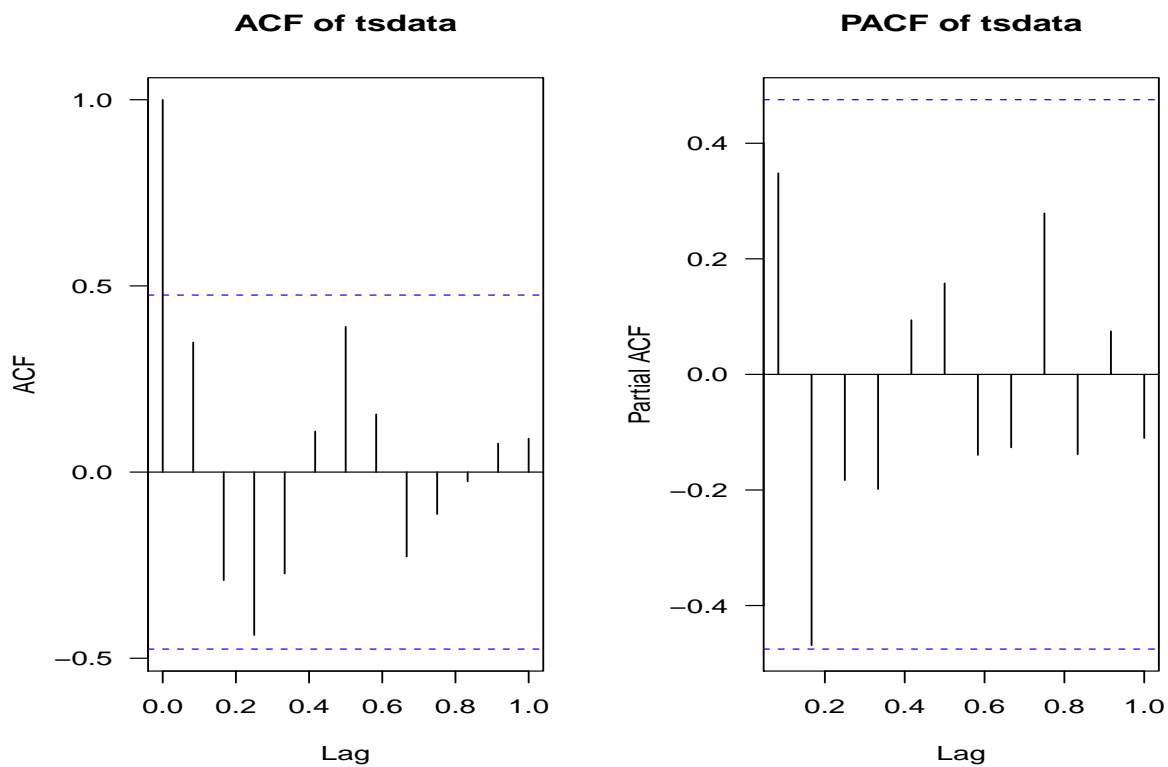


Figure 2: ACF and PACF plots for the Covid-19 deaths time series data for South Africa.

Table 2: Regression Coefficients.

Coefficients	Estimate	Std. Error	Z.value	p-value
ma1	0.565	0.193	2.922	0.004
Intercept	4977.198	1258.722	3.954	-0.0008

3.4. Model Validation

The linearity of the Q-Q plot and the bell-shaped histogram in Figure 3 suggests that the model residuals are normally and identically distributed. The ACF plot in figure has spikes which lie within the significant zone giving an implication that the residuals are random.

The p-value of the modified Ljung Box test statistics in Table 4 is greater than the significance value ($\alpha = 0.05$). This indicates that there are no serial correlations between lags and therefore, the residuals are distributed as white noise. We therefore conclude that the first order moving average is the perfect model for predicting the number of deaths resulting from Covid-19 pandemic in South Africa.

Table 3: Ljung Box test Statistics.

Test Statistics	Degrees of freedom	p-value
3.667	20	0.2998

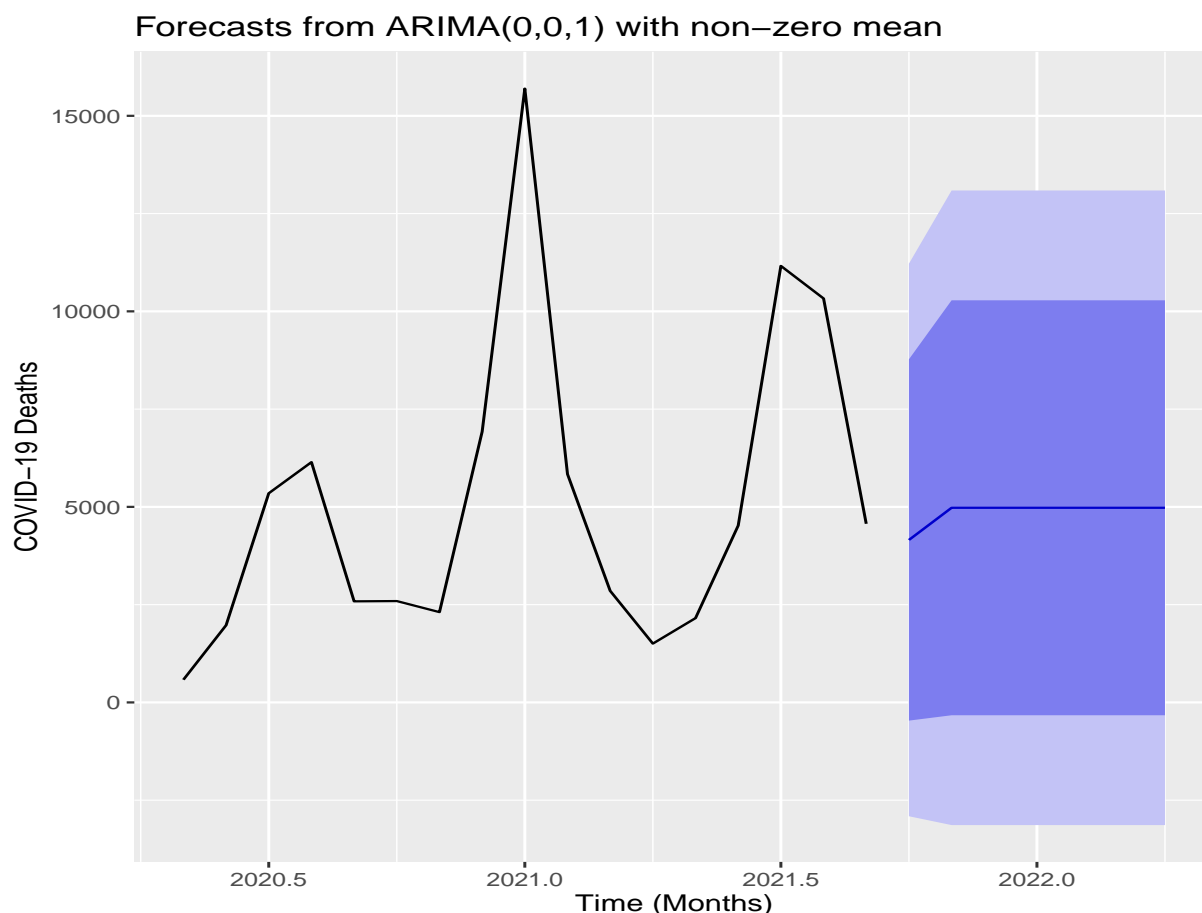


Figure 4: prediction plot for the Covid-19 Deaths in South Africa.

4. Conclusion

In this study, we applied the Autoregressive Integrated Moving Average (ARIMA) time series model to forecast the possible numbers of Covid-19 deaths in the Republic of South Africa for a period of 7 months running from October 1, 2020 to April 31, 2021. We adapted the Box-Jenkins' methodology to step-by-step to achieve the forecasting. Monthly data for Covid-19 deaths of the Republic of South Africa covering a period of 17 months (May 2020 to September 2021) was firstly plotted to check for the stationarity in the series. Augmented Dickey Fuller (ADF) test was conducted on the data clearly ascertaining that the data was stationary and thus differencing the data was unnecessary. We did plots on the Autocorrelation Function (ACF) and the Partial Autocorrelation Function (PACF) to respectively determine the Moving Average (MA) order and the Autoregressive (AR) order. Based on the ACF and PACF plots, the MA order was identified as one (1) whereas the AR order was identified as zero (0). Further, since the data was not differenced, then the differenced part (d) was taken as zero (0). This implied that the ARIMA(p,d,q) was identified to be given by MA(1) which is the first order moving average model.

The model was further fitted to the data using R statistical software and the parameters were estimated using the Maximum Likelihood Estimation algorithm implemented in R software. We applied the information criterion for the selection of the model where we identified the MA (1) as the best model based on both the Akaike Information Criterion (AIC) and the Bayesian Information Criterion (BIC). We next performed the validation test for the selected model where the model passed the test and therefore, it was considered for the prediction of the possible numbers of the Covid-19 deaths in South Africa. The forecasting was done at a 95% confidence interval and the predicted deaths were observed to slightly increase in the month of October but remained constant over the entire prediction period of time.

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